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# Coverage Control for Multi-Robot System Under Localization Uncertainty

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***Abstract*—This article proposes a multi-robot coverage control algorithm under positioning uncertainty to achieve coverage of specific areas in unknown environments. Using internal and external optimization methods to reduce the impact of positioning uncertainty and complete coverage tasks. The outer layer optimization uses momentum gradient descent to adjust the control input, while the inner layer optimization is based on extended Kalman filtering to dynamically update the robot state and reduce position uncertainty. The simulation experiments in this article demonstrate the effectiveness and robustness of this method in multi-robot collaboration, providing a new solution for robot deployment in unknown environments.**

***Index Terms*—Coverage control, Positioning uncertainty, Multi-robot systems**

## I. Introduction

In the past two decades, the application of mobile robots in area coverage control has garnered significant attention. Coverage control aims to optimally distribute a team of mobile robots within a designated task environment, ensuring the comprehensive sampling or monitoring of the entire area by maximizing the collective sensing capability of the robotic network. This technology finds its application across a broad spectrum, including surveillance [1], military protection [2], public safety [3], location services [4], and environmental sampling [5]. As detailed in the comprehensive review papers [6], [7], most typical algorithms of the coverage control problem are grounded in computational geometry that leverage Voronoi partitioning [8], [9], utilizing mobile robots equipped with exact self-localization and isotropic sensors (or communication ranges) to achieve static coverage tasks.

In multi-robot coverage control, robot localization uncertainty is an important issue. Localization uncertainty refers to the inability of robots to accurately determine their position in the environment, which can arise from factors such as sensor noise, environmental complexity, and unreliability of communication. Many existing methods improve the robustness of the system by introducing more complex algorithms when dealing with positioning uncertainty [10]. However, this often leads to a significant increase in computational complexity. The paper [11] proposes a robust and scalable multi-robot coverage method, which focuses on reducing the computational complexity of the system by optimizing the algorithm structure and distributed coordination mechanism. Currently, most research is conducted in idealized or structured environments, while real-world environments are typically dynamic and unstructured, lacking effective adaptation mechanisms, resulting in a significant decline in system performance in real-world environments [12]. Traditional methods use probabilistic models to handle uncertainty, but accurately quantifying and managing these uncertainties remains a challenge.

In response to the above issues, this article proposes a multi-robot coverage control framework that uses inner- and outer-layer optimization methods to achieve robot coverage tasks under localization uncertainty. The outer layer optimization uses momentum gradient descent to adjust control inputs to achieve coverage tasks, while the inner layer optimization uses an EKF-slam method based on extended Kalman filtering to dynamically update robot states and reduce positioning uncertainty. Through a series of simulation experiments, this paper

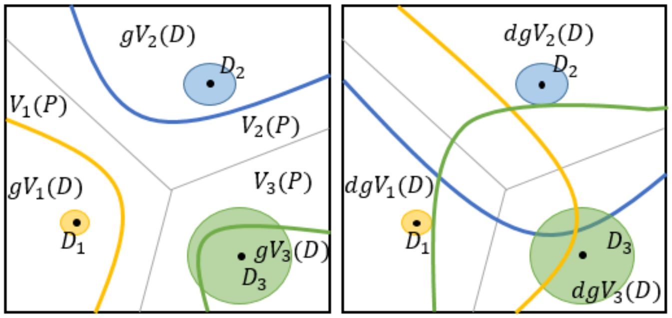


Fig. 1: The Guaranteed Voronoi diagram (left) and the Dual-Guaranteed Voronoi diagram (right) are depicted below. The true locations of the robots $P = \{p_1, p_2, p_3\}$ are indicated by the black dots. The uncertainty regions are represented by the circular areas $D_i$.

validates the effectiveness of the proposed method in complex unknown environments and demonstrates its significant advantages in multi-robot coverage control under positioning uncertainty.

## II. Preliminaries

### A. Voronoi Tesselation

The Voronoi partitions are a geometric segmentation method commonly used for region allocation in multi-robot coverage tasks. We let $\mathbb{R} \geq 0$ be the set of non-negative real numbers, respectively, and $\|\cdot\|$ be the Euclidean distance. Let $\boldsymbol{S}$ be a convex polygon in $\mathbb{R}^2$ and let $P = (p_1, \ldots, p_n)$ represent the locations of n robots. A partition of $\boldsymbol{S}$ is a collection of n polygons $W = \{W_1, \ldots, W_n\}$ with disjoint interiors whose union is $\boldsymbol{S}$. The Voronoi tesselation $V(P) = \{V_1, \ldots, V_n\}$ of $\boldsymbol{S}$ generated by the points $P = (p_1, \ldots, p_n)$ is defined as follows

$$V_i = \{q \in \boldsymbol{S} \mid \|q - p_i\| \leq \|q - p_j\|, \forall j \neq i\} \tag{1}$$

### B. Guaranteed and Dual-guaranteed Voronoi Tesselation

In order to adapt to the uncertainty of a dynamic environment and robot motion, we introduce guaranteed Voronoi partitions under the uncertainty of robot positioning. Let $D_i \in \boldsymbol{S}$ represent the uncertainty in the location of the $ith$ robot, i.e. $p_i \in D_i$, and let $D = D_1, \ldots, D_N$ denote the set of uncertainty regions for all robots. The $i$-th guaranteed Voronoi cell $gV_i(D)$ is defined as

$$gV_i(D) = \left\{ q \in \boldsymbol{S} \mid \underset{x \in Di}{max} \|q - x\| \leq \underset{y \in Dj}{min} \|q - y\|, \forall i \neq j \right\} \tag{2}$$

Complementary to the concept of guaranteed Voronoi cell, the $ith$ dual-guaranteed Voronoi cell $dgV_i(D)$ is defined as

$$dgV_i(D) = \left\{ q \in \boldsymbol{S} \mid \underset{x \in D_i}{min} \|q - x\| \leq \underset{y \in D_j}{max} \|q - y\|, \forall i \neq j \right\} \tag{3}$$

Fig. 1 shows examples of guaranteed Voronoi diagrams and dual-guaranteed Voronoi diagrams. Specifically, with knowledge of the guaranteed Voronoi cell $gV_i(D)$ and the dual-guaranteed Voronoi cell $dgV_i(D)$, one can derive an approximation of the centroid $c_V i$ with a quantifiable, bounded error. This approximation takes advantage of the following proposition from [13], Proposition 5.2.

Proposition 1: Let $U$, $V$, $W$ be three sets such that $U \subseteq V \subseteq W$. Then, for any density function $\phi$, the following holds.

$$\|c_V - c_U\| \leq dm(W) \left(1 - \frac{m_U}{m_W}\right) \tag{4}$$

In short, Proposition 1 means that all variables except $c_{V_i}$ are known. In subsequent optimization, the centroid error is used as part of the objective function to minimize the centroid error.

## III. Problem Formulation

We consider the multi-robot system composed of n robots moving in a 2-D plane $S$. Denote $p_i$ the position of $i$-th robot. The coverage control for the multi-robot system aims to enable robots to move in space to cover the target area. The goal of coverage is usually quantified by defining coverage performance indicators. The common solution is to minimize the position cost function $\mathcal{H}$ in the following form

$$\mathcal{H}(P, W) = \sum_{i=1}^{n} \int_{W_i} f(\|q - p_i\|) \phi(q) \, dq \tag{5}$$

$f(\|q - p_i\|)$ is the performance function between the position q and the robot distribution, and $\phi(q)$ is the weight distribution function of the target area. By dynamically adjusting the position of the robot, the coverage performance indicator $\mathcal{H}(P, W)$ is minimized to achieve optimal coverage of the target area.

In the presence of uncertainty, the motion model of $i$-th robot is the conventional state transition model with additive Gaussian noise.

$$p_i^{k+1} = f_i\left[p_i^k, u_i^k, w_i\right] \tag{6}$$

where $w_i \sim \mathcal{N}(0, R_w)$ with $R_w$ representing the noise covariance matrices of the process. We can represent it as a probability term $\mathbb{P}(p_i^{k+1} | p_i^k, u_i^k)$.

In addition, the observation model is

$$z_{i,j}^k = h(p_i^k, l_j, v_i) \tag{7}$$

where $v_i \sim \mathcal{N}(0, R_v)$, and $R_v$ represent the measurement noise covariance matrices. $l_j$ is the location of the landmark.

In the multi-robot system described in this article, we consider a general objective function involving coverage cost and localization uncertainty cost

$$J = c_{\mathcal{H}}(P, W) + \sum_{i=1}^{n} c_L(p_i^c) \tag{8}$$

$c_{\mathcal{H}}(P, W) = \mathcal{H}(P, W) = \sum_{i=1}^{n} \int_{W_i} \|q - p_i\|^2 \phi(q) \, dq$ is the cost of location coverage and $c_L(p_i^c)$ is the uncertainty of location cost of the robot.

We design to minimize this objective function, reducing errors caused by robots position uncertainty during the coverage process. Thus achieving optimal coverage effect.

## IV. METHODOLOGY

In this work, we will demonstrate how to incorporate guaranteed Voronoi partition and belief space planning into multi-robot coverage control tasks, significantly improving estimation accuracy and reducing the impact of localization uncertainty. Our method extends existing coverage task techniques and achieves high-precision multi-robot area coverage without absolute information sources that cause positioning uncertainty, such as reliable GPS, beacons, and known landmarks.

### A. Self-localization under uncertainty

In the case of GNSS-constrained coverage tasks, the first thing we need to solve is the problem of robot self-localization. Here, we use the Extended Kalman Filter (EKF) method to estimate the robot's state. In EKF slam, the first step is to update the state of each robot at each time step $k$. This includes using current control inputs $u_k$ to predict the state of the next time step

$$p_k = f(p_{k|k-1}, u_{k|k-1}) \tag{9}$$

Then, update the covariance matrix $M_k$ of the robot, which reflects the uncertainty of the state

$$M_k = F_{k|k-1}M_{k|k-1}F_{k|k-1}^{\mathsf{T}} + G_{k|k-1}QG_{k|k-1}^{\mathsf{T}} \tag{10}$$

among them, $F_{k-1}$ and $G_{k-1}$ are the $jacobian$ matrices of state and control. $Q$ and $R$ are the covariance matrices of process noise and observation noise, respectively. If new landmarks is observed during the robot's movement, the Kalman filter is used to update the state and covariance matrix. Update steps

$$z_k = h\left(p_{k|k-1}\right) + v_k \tag{11}$$

$$S_k = H_k M_{k|k-1} H_k^{\mathsf{T}} + R \tag{12}$$

$$K_k = M_{k|k-1} H_k^{\mathsf{T}} S_{k|k-1}^{-1} \tag{13}$$

$$p_k = p_{k|k-1} + K_k(z_k - h\left(p_{k|k-1}\right) \tag{14}$$

$$M_k = (I - K_k H_k)M_{k|k-1} \tag{15}$$

Finally, the updated robot state and covariance matrix were obtained by using the EKF method. Among them, $H_k$ is the $Jacobian$ matrix of the observation model, and $z_k$ is the observation vector.

### B. Guaranteed Voronoi partition

Our task is to perform coverage control while considering positioning uncertainty, where the robot coverage area $W_i$ is no longer deterministic. According to equation (5), the coverage optimization objective for a guaranteed Voronoi partition can be expressed as

$$\mathcal{H}\left(b\right) = \sum_{i=1}^{n} \int_{gV_i} \mathbb{E}[\|q - p_i\|^2]\phi\left(q\right)dq \tag{16}$$

As demonstrated in the paper [8], the (not necessarily unique) local minimum points of the locational optimization function $\mathcal{H}\left(P, gV(P)\right)$ are the centroids of their Voronoi cells

$$c_{gV_i} = \arg\min_{p_i} \mathcal{H}\left(P, gV(P)\right)$$

Although we have effectively reduced the impact of localization uncertainty through the use of a guaranteed Voronoi partition, we must also address the centroid error $\|c_V - c_{gV}\|$. After considering the centroid error, the objective function transforms as follows

$$\begin{aligned} J = &\mathbb{E}\left[\mathcal{H}(b(X) + c_{uncert}\left(b(X)\right))\right] \\ &+ \|c_V - c_{gV}\| \end{aligned} \tag{17}$$

The robot moves towards the guaranteed Voronoi partition in each iteration, which reduces the value of the optimization function. As noted in reference [8], the coverage cost function exhibits a monotonically decreasing behavior. In optimization algorithms such as filtering, after multiple iterations and updates, the covariance matrix stabilizes, and the uncertainty error tends towards a constant value.

By further combining Proposition 1, the upper bound of the objective function can be obtained as

$$\begin{aligned} J - &\mathbb{E}\left[\mathcal{H}(b(X) + c_{uncert}\left(b(X)\right))\right] \\ &\leq d_{centroid} \cdot \left(1 - \frac{m_g V}{m_{dg} V}\right) \end{aligned} \tag{18}$$

This we conclude that the objective function is bounded. Finally, as the robot approaches the centroid of the guaranteed Voronoi cell, the objective function converges, minimizing both the coverage cost and the centroid error, leading to an optimal and stable configuration.

## V. SIMULATION AND RESULTS

The algorithm has been validated through simulations involving ground-based motion robots and implemented using MATLAB. The simulation environment is a two-dimensional plane, typically modeled as a rectangular region, with multiple landmarks strategically placed across the plane for robots to observe during movement. In the experiment, the robots are randomly distributed across the plane. The density function used for this coverage task is a two-dimensional Gaussian distribution, which represents varying coverage requirements across different locations. Higher-density regions indicate a need for more intensive coverage, while lower-density areas indicate lower coverage needs.

Compared with other algorithms [8], using the same Gaussian density function with consistent parameters yields a better monotonic convergence of the cost function, as shown in Fig. 2.

The coverage task function is $\mathcal{H}\left(P\right)$, therefore, it is an evaluation metric for the overall coverage effect. If it converges and stabilizes through iteration, it indicates that the robot has completed the coverage of the area. When using the Gaussian distribution density function, the coverage cost function is shown in Fig. 3.

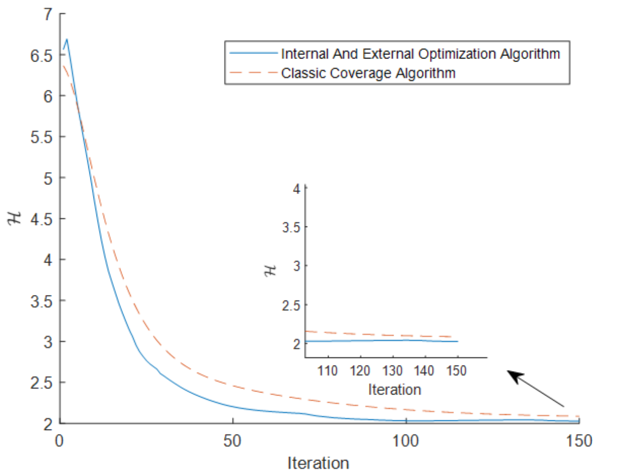


Fig. 2: The experiment shows that compared with other algorithms, this algorithm can converge faster and the final value is smaller for the coverage cost function.

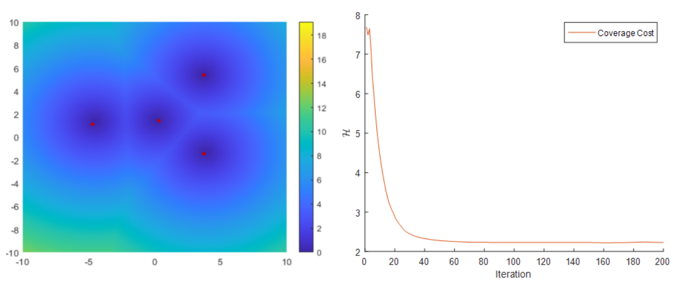


Fig. 3: Use Gaussian distribution density function $\phi(x, y) = exp\left(-\frac{(x-2)^2+(y-2)^2}{25}\right)$.

The path of the robots during the coverage task is shown in Fig. 4. The observation of landmarks during robots' movement can effectively reduce positional uncertainty.

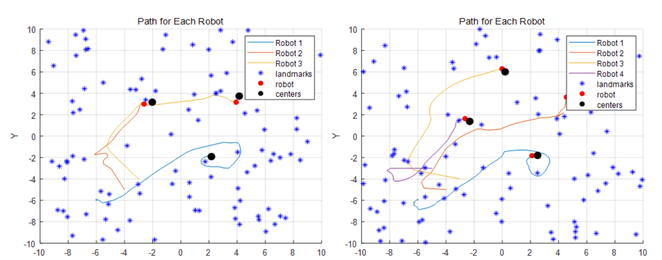


Fig. 4: The diagram shows the path of the robots during the coverage task.

## VI. Conclusion

This article proposes a new method for optimizing multi-robot coverage control under positioning uncertainty. Our method effectively divides the target area into conservative Voronoi regions, allowing each robot to focus on covering its designated area. Robots iteratively adjust their positions to optimize coverage. Optimizing the internal and external layer methods enables each robot to manage positioning uncertainty during motion and achieve coverage tasks. The proposed method has been tested in various scenarios, including using different numbers of robots, covering different distribution density functions in different areas, and comparing with other methods. The experimental results show that our method can effectively perform multi-robot coverage control under positioning uncertainty. Specifically, the dual-layer optimization framework enables robots to minimize positioning uncertainty and reduce overall coverage costs, demonstrating its effectiveness in unknown and complex environments.

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