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Optimal Placement of Positioning Service for Multi-Robot System using Relative Angle Measurement

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Abstract: In recent years, active positioning services based on cooperative localization using Multi-Robot System (MRS) have garnered significant attention as an effective method for positioning in Global Navigation Satellite Systems (GNSS)-denied environments. These services initially determine the precise location of the MRS using external positioning systems or onboard odometry, then propagate error location information about the target through sensors carried by the MRS. The geometric configuration of the MRS affects the quality of active positioning services. Current research primarily focuses on optimizing the deployment of wireless sensors (e.g. radar), with a limited exploration into the deployment of more accurate and cost-effective bearing-measurement sensors. This paper utilizes MRS to acquire bearing information of users and employs Position Dilution of Precision (PDOP) to assess localization accuracy for individual users. By integrating PDOP across the entire mission space, the optimal active localization service problem is formulated as an MRS layout optimization issue, with solutions proposed for scenarios involving obstacles. Consequently, the control scheme based on gradient ascent is developed, with extensive simulations demonstrating the efficacy of the proposed control law.

Keywords: Active positioning service, optimal placement, PDOP field coverage, bearing-measurement sensors.

1. INTRODUCTION

The ability to obtain precise, stable, and cost-effective positioning services that can maximally fulfill users' localization requirements and provide them with accurate location information plays a crucial role in the advancement of industrial progress in society as shown in Coffie (2020). However, as the core of modern positioning and navigation technology, GNSS has, in recent years, faced when addressing many sophisticated positioning tasks, such as autonomous driving and precision agriculture limitations, due to its susceptibility to signal interference and the constrained accuracy it offers in complex environments. Therefore, active positioning services, leveraging the flexible deployment of MRS and utilizing onboard sensors for cooperative localization, can offer improved positioning services to users in GNSS-constrained environments. They have garnered increased attention, such as Rohm et al. (2014); Zangenehnejad and Gao (2021).

Active positioning services typically utilize MRS to provide users with precise location information based on their positioning needs within the mission space through cooperative localization that was researched in Patwari et al. (2005); Huang et al. (2017); Gao et al. (2024). Firstly, the MRS can determine its own position using onboard odometry or external high-precision positioning systems. Then, through the use of sensors or other devices, it performs cooperative localization and communication with the target. Each mobile robot acts as the positioning anchor, propagating its noise-contaminated measurements of the user's position. Over the past few decades, much research on cooperative localization has revealed that the proper geometric configuration of sensors can significantly impact localization accuracy. To enable MRS to provide optimal positioning services within the mission space by active positioning service, it is essential to dynamically deploy each mobile robot to achieve the optimal geometric configuration ultimately.

Recently, an extensive study of improving the accuracy of cooperative localization systems by optimizing the geometric configuration of sensors has been conducted. Existing research has predominantly focused on serving a single target. For example, Khalife and Kassas (2018); Sadeghi et al. (2020, 2021) investigated optimizing the geometric configuration of MRS-equipped wireless sensors to provide optimal positioning services for a single localization target. The wireless sensors onboard the MRS can obtain distance and bearing information to the target through measurements. In Xu (2020), each sensor can obtain distance information between itself and a single target. The Fisher Information Matrix (FIM) is used as a metric to evaluate localization accuracy, enabling the determination of the optimal sensor placement under various environmental constraints. Sadeghi et al. (2021) adopts the bearing measurement strategy through the Cramér-Rao lower bound (CRLB) to obtain the optimal geometric configuration of sensors.

However, research focused solely on optimizing sensor deployment for a single target is insufficient to meet mod-

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ern applications' complex localization demands. Therefore, more advanced active positioning services must be capable of accommodating large-scale localization needs within the mission space, thereby serving targets with localization demands that may arise at any time. In this regard, Merriaux et al. (2017) has conducted research on multi-sensor optimization deployment for localization services targeting multiple objectives. However, there are limitations when the targets are dynamic and change their positions. To address this, Zhang et al. (2021) further refined the active positioning service problem by integrating it into the framework of coverage control. By measuring the distance between the targets and the MRS, a novel distributed control strategy was proposed, enabling optimal MRS deployment for dynamic multi-target users within the mission space through gradient-based computation methods. The aforementioned work can serve users with different localization needs effectively. However, these studies are based on ideal conditions. In real-world mission space, immovable obstacles such as workbenches or fixed equipment are inevitable and can significantly impact the robots' ability to observe users.

Coverage control refers to the process of defining an objective function towards control objectives and utilizing a multi-agent system, such as MRS, to achieve the final target network within the task space through appropriate control methods, the researches about coverage control can be seen in Huang et al. (2018); Wang and Hussein (2010). Inspired by Zhang et al. (2021), this paper concentrates on the optimal placement of MRS, which is equipped with bearing measurement sensors for providing dynamic-dispersed users in the mission with the active positioning service. Besides, in this paper, we use PDOP, and the concept can be seen in Cellmer (2004) as the accuracy metric. a gradient-based control law could be obtained by computing the PDOP field to drive the MRS update whose pose and optimize the geometric configuration. Consequently, MRS provides the optimal localization services for users within the mission space.

The main contributions of this work are:

(1) This paper proposes the optimal placement problem into a coverage control problem for active positioning service using bearing-measurement sensors in the mission space that contains the obstacles.
(2) We propose a strategy for updating the poses of the MRS using a centralized control law by PDOP field and numerical-gradient-ascent algorithm. Extensive simulation experiments have demonstrated the effectiveness of our control law.

2. PROBLEM FORMULATION

In the 2-D x-y mission space $\boldsymbol{S} \subset \mathbb{R}^2$ where GNSS is limited, we deploy a set of mobile robots to provide active positioning for all potential users by relative bearing measurement(e.g. cameras). As is shown in Fig. 1, the pose of i-th mobile robot can be expressed as $\boldsymbol{M}_i = [x_i, y_i, \psi_i]^T$, where $[x_i, y_i] \in \boldsymbol{S}$ is the position of i-th mobile robot, ψ_i stand for the heading angle of the mobile robots whose value is the yaw angle with regard to x axis. we denote the joint pose vector for all mobile robots as $\boldsymbol{M} = [\boldsymbol{M}_1, \boldsymbol{M}_2, \cdots, \boldsymbol{M}_N]^T$.

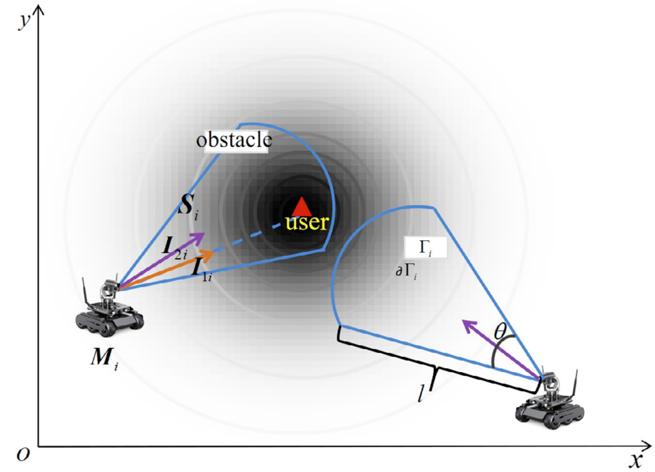


Fig. 1. 2-D space with notation definitions.

Constrained by the limited horizontal field of view (FOV) of the bearing measurement, we use a new coverage model as the circular sector $\boldsymbol{S}_i \subset \boldsymbol{S}$ whose center is the position of i-th mobile robot as is shown in Fig. 1. In practical applications, the coverage area of a bearing measurement sensor is a sector. The size of the coverage area for the i-th mobile robot is typically determined by the maximum measurement range l_i and the maximum horizontal FOV angle of the bearing measurement sensor β_i it carries. In this paper, we use the same mobile robots, i.e., $l_1 = l_2 = \cdots = l_i$, $\theta_1 = \theta_2 = \cdots = \theta_i$.

There are various methods to evaluate the localization accuracy that MRS provide for users; inspired by Zhang et al. (2021), we use the PDOP as our accuracy index. We consider that the user $\boldsymbol{u}$ is in the mission space, as is shown in Fig. 1. We denote $\boldsymbol{N}_r^k = \{\boldsymbol{M}_i | \boldsymbol{r} \in \boldsymbol{S}_i\}$ as the observed set of mobile robots from the user $\boldsymbol{r}$ at time step k, and the N_r^k stand for the size of set $\boldsymbol{N}_r^k$. the unit direction vector $\boldsymbol{I}_i$ from the user $\boldsymbol{r}$ to the i-th mobile robots is

$$\boldsymbol{I}_{1i} = \frac{\boldsymbol{r} - \boldsymbol{M}_i}{\|\boldsymbol{r} - \boldsymbol{M}_i\|_2} \tag{1}$$

where $\|\boldsymbol{r} - \boldsymbol{M}_i\|_2$ represents the distance from $\boldsymbol{r}$ to $\boldsymbol{M}_i$ in the mission space. Additionally, we introduce the $\boldsymbol{I}_{2i}$ vector to represent the directional vector aligned with the orientation of the i-th mobile robot. Thus, the set $\boldsymbol{N}_r^k$ can also be expressed as:

$$\boldsymbol{N}_r^k = \left\{\boldsymbol{M}_i | \langle \boldsymbol{I}_{1i} \cdot \boldsymbol{I}_{2i} \rangle < \tfrac{\theta}{2} \cap \|\boldsymbol{r} - \boldsymbol{M}_i\|_2 \le l\right\} \tag{2}$$

where $\langle \boldsymbol{I}_{1i} \cdot \boldsymbol{I}_{2i} \rangle$ is the angle between $\boldsymbol{I}_{1i}$ and $\boldsymbol{I}_{2i}$.

$\langle \boldsymbol{I}_{1i} \cdot \boldsymbol{I}_{2i} \rangle = \cos^{-1}(\boldsymbol{I}_{1i} \cdot \boldsymbol{I}_{2i})$. The Jacobian matrices $\boldsymbol{J}$ of the N_r^k mobile robots bearing-measurement errors evaluated at the true user position $\boldsymbol{r}$ represents

$$\boldsymbol{J} = \begin{bmatrix} -\frac{y_r - y_1}{d_1^2} & \frac{x_r - y_1}{d_1^2} \\ -\frac{y_r - y_2}{d_2^2} & \frac{x_r - y_2}{d_2^2} \\ \vdots & \vdots \\ -\frac{y_r - y_{N_r^k}}{d_{N_r^k}^2} & -\frac{y_r - y_{N_r^k}}{d_{N_r^k}^2} \end{bmatrix} \tag{3}$$

In this addition, the performance of localization information provided through the MRS could be expressed by the PDOP, which has a close connection with $\boldsymbol{J}$ and can stand for

$$PDOP(\boldsymbol{r}, \boldsymbol{M}) = tr\left((\boldsymbol{J}^T\boldsymbol{J})^{-1}\right) \tag{4}$$

where $tr(\cdot)$ represents the compute of the matrix's trace.

Remark 1. The PDOP is a crucial metric for assessing the impact of sensor geometric deployment on the accuracy of localization systems. Its value reflects the extent to which the geometric arrangement of sensors amplifies the error in calculating the user's actual position. Therefore, we typically aim to find smaller PDOP values as a means to optimize the deployment of MRS.

Instead of providing positioning service for a single user, in this paper, the active positioning service system is dedicated to serving all users that may appear in the mission space, a density function $\mu(\boldsymbol{r})$ that is positive and finite could be adopted to denote the possibility of the user appearing in a specific position. Then we use a Gaussian density function to express the mission space, which can be described follows:

$$\mu(\boldsymbol{r}) = \exp(-\beta \|\boldsymbol{r} - \boldsymbol{r}_d\|_2) \tag{5}$$

where $\|\boldsymbol{r} - \boldsymbol{r}_d\|_2$ is the distance from user $\boldsymbol{r}$ to the center of density function $\boldsymbol{r}_d$, and β is the decay parameter of the Gaussian density function.

Remark 2. The mission space is described using a Gaussian density function, allowing it to be viewed as a hotspot region. The center of the density function represents the center of this hotspot, indicating the highest probability of user presence. As the distance from the center increases, the probability of user presence decreases.

In practical applications, it is often inevitable for obstacles to appear within the mission space (e.g. immovable workbenches). If these obstacles are located between the sensor and the user, they can directly impede the propagation of relative distance and bearing information between the sensor and the user. In this paper, as illustrated in Fig. 1, we denote the i-th obstacle as Γ and the boundary of the i-th obstacle as $\partial\Gamma_i$. Since the user cannot be located within an obstacle, we set the density function value of the task space occupied by obstacles to zero. Then the density should be expressed as:

$$\mu(\boldsymbol{r}) = \begin{cases} 0, & \boldsymbol{r} \in \boldsymbol{\Gamma}_i \cup \partial\boldsymbol{\Gamma}_i \\ \exp(-\beta \|\boldsymbol{r} - \boldsymbol{r}_d\|_2, & \boldsymbol{r} \notin \boldsymbol{\Gamma}_i \cup \partial\boldsymbol{\Gamma}_i \end{cases} \tag{6}$$

In addition, We use ζ_i to denote the line connecting the user and the i-th mobile robot. equation (2) should be redefined as follows.

$$\boldsymbol{N}_r^k = \left\{\boldsymbol{M}_i | \langle \boldsymbol{I_{1i}} \cdot \boldsymbol{I_{2i}} \rangle < \tfrac{\theta}{2} \cap \|\boldsymbol{r} - \boldsymbol{M}_i\|_2 \leq l, \zeta_i \cap \partial\Gamma_i = \varnothing \right\} \tag{7}$$

then at time step k, we can compute the PDOP by (4)

Inspired by Zhang et al. (2021), we change the active positioning service problem into maximizing the next objective placement optimization function:

$$\mathcal{F}(\boldsymbol{M}) = \int_{\boldsymbol{S}} \mu(\boldsymbol{r}) h(\text{PDOP}(\boldsymbol{r}, \boldsymbol{M})) d\boldsymbol{r} \tag{8}$$

where $h(\cdot)$ is a continuous and monotonically decreasing function; therefore, smaller PDOP correspond to larger $h(\cdot)$ values which also called the generalized PDOP value.

In this paper, we aim to identify an effective control law for each mobile robot, enabling the MRS to achieve an optimal deployment that enhances the overall performance of active localization services. This can be accomplished by solving the following optimization problem:

$$\begin{aligned} &\max_{\boldsymbol{M}} \int_{\boldsymbol{S}} \mu(\boldsymbol{r}) h(\text{PDOP}(\boldsymbol{r}, \boldsymbol{M})) d\boldsymbol{r} \\ &\text{s.t.} \quad [x_i, y_i] \in \boldsymbol{S}, \quad i = 1, 2, \cdots, N \end{aligned} \tag{9}$$

3. CONTROL METHOD

To utilize the MRS equipped with bearing-measurement sensors for providing active positioning services to users, it is necessary to use more than two sensors to estimate the user's positional information. If the user is within the measurement range of only one mobile robot, i.e. $N_r^K = 1$ so that $\det(\boldsymbol{J}^T\boldsymbol{J}) = 0$, we have no method to compute the PDOP in (4), in order to solve this problem, We can isolate these special cases by redefining the optimization function. In this section, we redefine the previous objective optimization function and design a centralized control law based on gradient ascent.

3.1 Redefine the optimization function

We propose a new function $\bar{h}(\cdot)$, if the user is observed by only one mobile robot or all the $\boldsymbol{I}_{1i}$ are parallel, the PDOP cannot be computed, because $\det(\boldsymbol{J}^T\boldsymbol{J}) < 0$. Consequently, no method can be adopted to optimize the active localization service under such conditions. Inspired by Kang et al. (2017), we can assign a very small value to $\bar{h}(\cdot)$ when $\det(\boldsymbol{J}^T\boldsymbol{J}) < 0$. This approach allows us to set up a gradient ascent-based control law later, allowing the MRS to move toward the center of the density even when PDOP cannot be calculated in the initial state, where is the location with the highest probability of user presence. the $h(\cdot)$ will be designed as follow:

$$\bar{h}(\cdot) = \begin{cases} 0, & \text{if } N_r^k = 0 \\ a, & \text{if } N_r^k = 1 \\ b, & \text{if } N_r^k \geq 2 \text{ but} \det(\boldsymbol{J}^T\boldsymbol{J}) = 0 \\ h(\cdot), & \text{otherwise} \end{cases} \tag{10}$$

where a and b are two positive constants, and they meet the relationship as $a < b$ because if the user is observed by more mobile robots, the error in estimating their position will be reduced, thereby decreasing the PDOP.

Remark 3. In this paper, we hope that $h(\cdot) : [1, 2] \to \mathbb{R}$ which is continuous and monotonically decreasing. Inspired by Zhong and Cassandras (2011), the role of $h(\cdot)$ is to converge higher or lower gains into a rational reward range. If the user is too close to or too far from the mobile robot, the calculated PDOP may exhibit extreme values, either too small or too large. We designed $h(\cdot)$ not only to represent the MRS active positioning service problem using a unified mathematical model but also to ensure that the calculated localization accuracy metrics converge within a reasonable range. This prevents the phenomenon of localization service resources from being concentrated entirely in a single location.

3.2 Gradient Derivation and Control Law

In the previous subsection, we replace $h(\cdot)$ with $\bar{h}(\cdot)$ so that the optimization problem in (9) should be redefine as

$$\begin{aligned} \boldsymbol{M} &= \arg\max_{\boldsymbol{M}} \int_{\boldsymbol{S}} \mu(\boldsymbol{r})\bar{h}(\mathrm{PDOP}(\boldsymbol{r},\boldsymbol{M}))d\boldsymbol{r} \\ \text{s.t.} &\quad [x_i, y_i] \in \boldsymbol{S}, \quad i = 1, 2, \cdots, N \end{aligned} \tag{11}$$

Inspired by coverage control problems in Kang et al. (2017), in this paper, we formulate a gradient-based control method to solve the optimization problem in (11), on the basis of the gradient ascent rule, the pose of the i-th mobile robot at the next time step is given by:

$$\boldsymbol{M}_i^{k+1} = \boldsymbol{M}_i^k + [\lambda\frac{\partial \mathcal{F}(\boldsymbol{M})}{\partial x_i}, \lambda\frac{\partial \mathcal{F}(\boldsymbol{M})}{\partial y_i}, \lambda_0\frac{\partial \mathcal{F}(\boldsymbol{M})}{\partial \psi_i}] \tag{12}$$

where k is the time step, λ is the step size of updating position, λ_0 is the step size of updating orientation. Inspired by the problems about optimal sensor positioning in Kang et al. (2017), the MRS can adjust whose pose by control Algorithm. To solve the gradient in (12) easily, we use the forward difference to approximate the partial derivative with respect to pose, which can be denoted as:

$$\begin{aligned} \frac{\partial \mathcal{F}(\boldsymbol{M})}{\partial x_i} &= \frac{\mathcal{F}(x_i+\Delta x, y_i, \psi_i) - \mathcal{F}(x_i, y_i, \psi_i)}{\Delta x} + \sigma \\ \frac{\partial \mathcal{F}(\boldsymbol{M})}{\partial y_i} &= \frac{\mathcal{F}(x_i, y_i+\Delta y, \psi_i) - \mathcal{F}(x_i, y_i, \psi_i)}{\Delta y} + \sigma \\ \frac{\partial \mathcal{F}(\boldsymbol{M})}{\partial \psi_i} &= \frac{\mathcal{F}(x_i, y_i, \psi_i+\Delta \psi) - \mathcal{F}(x_i, y_i, \psi_i)}{\Delta \psi} + \sigma \end{aligned} \tag{13}$$

where $\Delta x, \Delta y, \Delta\psi$ are minute perturbations that are employed to emulate the analytical process of partial differentiation, yielding computationally feasible numerical gradient. σ is a value that meets Brownian motion, which can help to avoid traps of local optima and enable a more comprehensive search for the global optimum.

Given the derivation in the previous paper, we can design the main control law for updating each robot's pose. Considering our objective of rapidly achieving a globally optimized result, we propose a centralized control method. Within the mission space, a central controller is deployed as the core of global decision-making. It is assumed that all mobile robots are equipped with communication devices. The central controller collects pose information from all mobile robots via a real-time communication network. Based on the calculated gradient direction, it determines the new expected poses for the mobile robots and sends movement commands (including angular and linear velocities) to guide them to the specified poses. This process iterates, with gradient calculations continuing to control the movement of the MRS, ultimately achieving the optimal configuration for active localization services.

In order to compute the integrals in (11), we use a numerical integration approach to approximate integral operation. The core of numerical integration involves first discretizing the task space, followed by summing the integrated. In this paper, we define an adjustable resolution parameter G which is tunable for the discretization of the mission space. generally, the control method of the i-th mobile robot at time step k can be shown in Algorithm 1.

Algorithm 1 Controller of i-th Robot at time step k

Require: λ, λ_0, $\boldsymbol{M}_i^k$, G
Ensure: $\boldsymbol{M}_i^{k+1}$

1: Discretize the mission into G^2 meshes, and the central point M_c stands for the mesh.
2: Determine the observed robots set of each mesh $\boldsymbol{N}_j^k$ $j = 1, \cdots, G^2$, with the method described in (7).
3: For each point $\boldsymbol{M}_c$ in the mission space, compute the optimization function (11) by accumulating the product value of the density value of each mesh, $\bar{h}$(PDOP) and the size of the mesh.
4: Derive the gradient by (13).
5: Update the $\boldsymbol{M}_i^{k+1}$ through the λ and λ_0 using (12).
6: **Return** $\boldsymbol{M}_i^{k+1}$.

4. SIMULATION

To validate the rationality and effectiveness of our proposed control method, we designed two sets of simulation experiments. In the first tests, we specified the density function of the mission space and the step size for updating the poses of the MRS, thereby verifying the feasibility and correctness of the centralized control scheme. In the second set of simulations, we introduced a variable density function within the task space to simulate dynamically changing users. This demonstrated that our control method can provide proactive positioning services for users with dynamic variations.

Let's denote the mission space $\boldsymbol{S}$ as $\{[x, y] | 0 \leq x, y \leq 100\}$, the coverage model used in this paper is a sector whose maximum measurement is 40, the maximum field of view angle is 60°, and we propose the function $\bar{h}(.)$ which can be express as:

$$\bar{h}(x) = 1 + \frac{1}{e^{\alpha x}} \tag{14}$$

where α is an adjustable parameter introduced to slow down the rate of decline in the PDOP, thereby preventing a sharp increase in the value of the objective function, we have chosen $\alpha = 0.001$.

The Gaussian density function (6) is used in this section to stand for the mission space $\boldsymbol{S}$, and we choose the decay parameter is 1 in the following simulation experiments, the center of density function can be changed to represent different users in the mission space. Theoretically, the resolution G affects the accuracy of numerical integration. A larger G enhances computational accuracy but also increases computational complexity. We have chosen a compromise by fixing G at 100 and proceeding with the subsequent simulation experiments. Besides, we set the pose updating size $\lambda = 0.3$ and $\lambda = 2°$. Lastly, we use $\frac{\sigma}{m}$ as the value of a random term that satisfies Brownian motion to avoid the influence of overly large random values on the search for the optimal layout; m is the maximum number of iterations.

4.1 MRS Serving a Static User

In this set of simulation experiments, we utilized four mobile robots with random initial poses to provide active positioning service for a mission space with a density center at $[50, 50]$. In this section, we conducted 80 experiments,

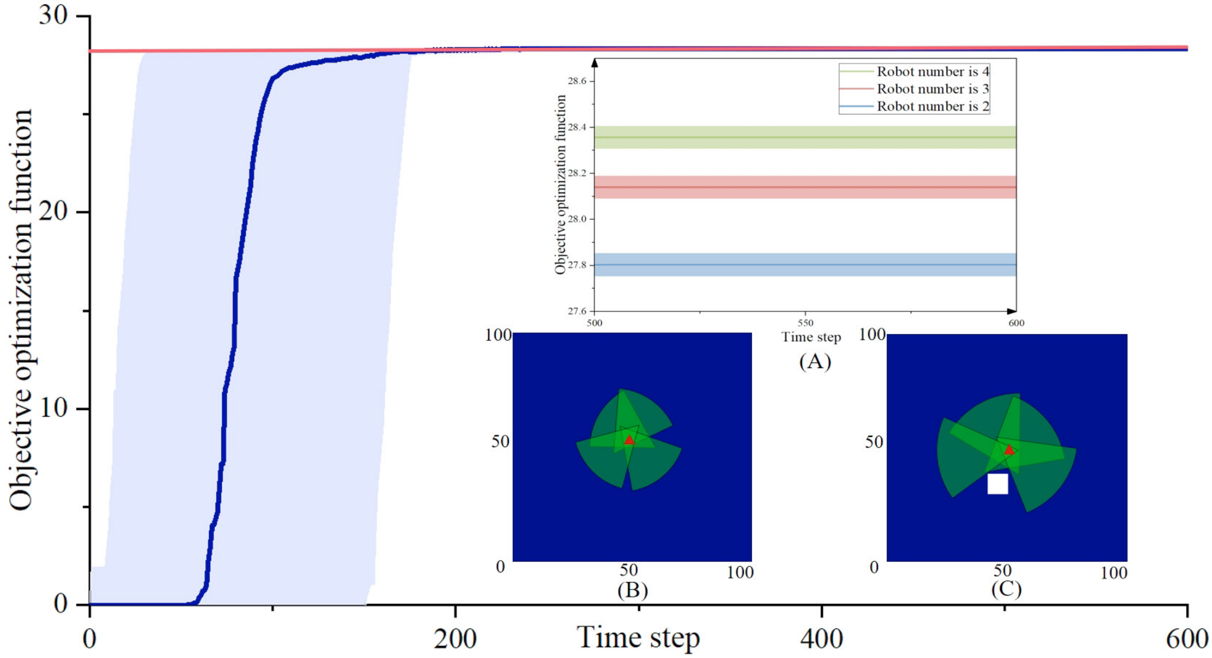


Fig. 2. Results for four mobile robots serving a static user, (A) represents the performance of different active positioning services under varying numbers of mobile robots. (B) illustrates the optimal MRS placement in the absence of obstacles, while (C) shows the optimal MRS deployment in the presence of obstacles which is shown as a white rectangle.

each consisting of 600 time steps. Before each experiment, we randomly generated the poses of the four mobile robots. Each robot updated its pose according to Algorithm 1.

The results of the simulation experiments are shown in Fig. 2. After conducting 80 experiments, we selected the MRS configuration from the experiment that yielded the highest value of the objective optimization function as the optimal configuration. Its coverage performance is represented by the red line in Fig. 2, and the distribution of the mobile robots is depicted in Fig. 2(B). Notably, the results of all 80 experiments converged to outcomes close to the global optimal solution, indicating that with a reasonable adjustment of the initial state, we can achieve optimal active positioning service. Additionally, we performed the localization service task using 4, 3, and 2 mobile robots by randomly assigning initial states and following Algorithm 1. The final results of the objective optimization function are shown in Fig. 2(A), demonstrating that a greater number of mobile robots can provide better active localization services performance. Finally, we validated the performance of our control method in the presence of obstacles. We introduced an impediment to the mission space with a center at $[50, 30]$, and the width and height are 10. Four mobile robots were randomly deployed and updated their poses continuously according to Algorithm 1. The placement of the resulting MRS is shown in Fig 2(C). This indicates that our control method can still deliver effective active localization services even in the presence of obstacles.

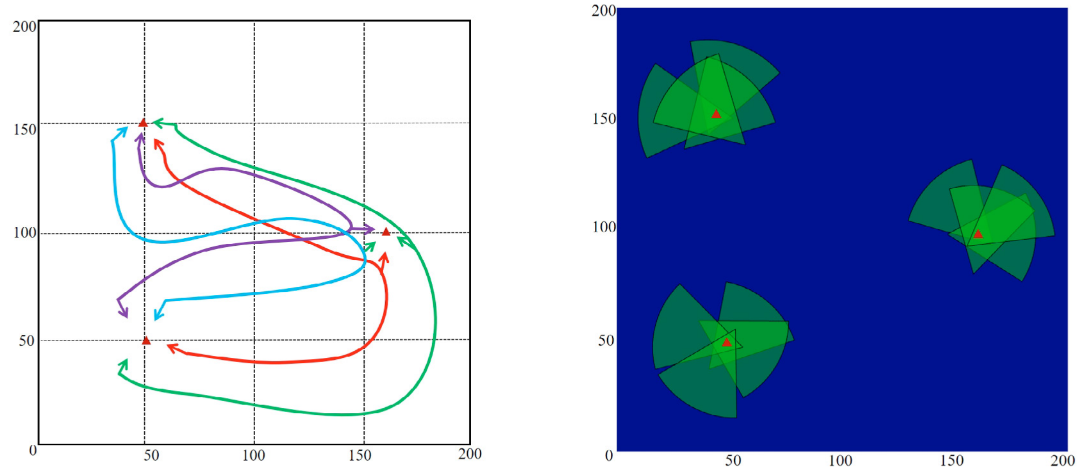
4.2 MRS Serving Dynamic User

In the last simulation experiment, we evaluated the provision of dynamic active positioning services in the mission space using four mobile robots over a sequence of time periods. Initially, a density center point was specified for the MRS to provide localization services. The sequence of these centers is $\boldsymbol{r}_{d1} = [50, 50]$, $\boldsymbol{r}_{d2} = [160, 100]$, $\boldsymbol{r}_{d3} = [50, 150]$, every time period lasts 600 steps. The demonstration is presented in Fig. 3, illustrating that our control method can track changes in the density center within the mission space, thereby enabling active positioning services for dynamic users.

5. CONCLUSION

In this paper, we propose a centralized solution that guides the MRS to provide active positioning services in a 2D mission space through bearing measurements. The performance of the localization service is defined by the PDOP field. We derive the PDOP field gradient for each mobile robot using numerical gradients, thereby applying a gradient ascent algorithm to update the pose of MRS. Through simulation experiments, our designed control method is capable of operating in mission spaces with obstacles and can track dynamically users.

However, our designed control method has several significant limitations that cannot be overlooked. Firstly, our control law is centralized, which, although advantageous in terms of response speed, poses a risk of complete system failure if a single node malfunctions. Therefore, in the future, we aim to design a distributed control law to enhance the robustness and scalability of the entire proactive localization service system. Additionally, our simulation experiments are conducted under ideal conditions, whereas in practical applications, collisions with obstacles and among mobile robots themselves may occur. Addressing collision avoidance will also be a key focus of our future research.



(a) Trajectories for four mobile robots. Different colors represent different mobile robots, and the arrows indicate the orientation of the mobile robots upon reaching different density centers. The starting point of each arrow represents the position of the mobile robot.

(b) The stable states of the four mobile robots when they reach different density centers are represented, with the red triangle indicating the density center.

Fig. 3. Simulation for the time-varying density function mission space over 1800 steps, whose centers are changed by 600-time steps. The order of these centers is $\boldsymbol{r}_{d1} = [50, 50]$, $\boldsymbol{r}_{d2} = [160, 100]$, $\boldsymbol{r}_{d3} = [50, 150]$. These centers are also the points where the user has the highest probability of appearing.

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