Finite-region H_∞ filtering of 2D Markov jump systems with deception attacks

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Abstract

In this paper, we study the finite-region H_∞ filtering problem for two dimensional (2D) Markov jump systems (MJSs) subject to deception attacks. First, the effect of deception attacks are considered and the filtering error is modeled. The sufficient conditions to ensure the stability of the filtering performance are obtained by using the Lyapunov function and linear matrix inequalities. Finally , the effectiveness of the method is verified by a numerical example on Darboux equation.

CCS Concepts

• **Networks** → *Mobile and wireless security*.

Keywords

finite-region, 2D MJSs, deception attacks,

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1 Introduction

With the rapid development of information technology, networked control systems have been widely used in various fields, such as smart grids[1], driverless cars[2], industrial automation[3], and medical equipment[4]. However, due to the openness and complexity of networks, networked control systems are vulnerable to various attacks, especially deception attacks. These attacks attempt to mislead the system's decision-making process by forging or tampering with sensor data, which leads to system performance degradation or even failure. Researches on deception attacks have attracted much attention from scholars. Reference[5] proposed network deception attacks method that employs a Hidden Markov Model to predict the most likely sequence of attack paths and deploys decoy nodes along the predicted paths. Reference[6] studied the scheduling of deception attacks for a class of discrete-time systems with attack detection capabilities. Reference[7] affected by deception attacks, the problem of state estimation for Markovian leapfrog systems based on neural networks is explored. Reference[8] studied the sliding mode control of half-Markov jump cyber-physical systems under randomly occurring deception attacks. Despite all the results obtained, the problem of deception attack on control systems is still a hot research issue.

2D systems have important applications in many areas of engineering and science. In recent years, with a deeper understanding of the system and a deepening of the breadth of practical applications, some of the more complex situations are difficult to describe using 1D systems. Scholars have therefore begun to explore 2D systems.Cheng *et al*. [9] constructs a class of 2D MJSs based on the roesser model and investigates its finite-region H_∞ asynchronous control problem. Shen *et al*. [10] designed asynchronous fault detection filters and explored the problem of fault detection in 2D MJSs with roesser model. Reference[11] studied the asynchronous event-triggered control problem for 2D MJSs in the presence of random packet loss in the network, which effectively reduces the pressure on network communication and ensures the stability and performance of the closed-loop system. However, Research on the control design problem for 2D MJSs are still scarce and need further in-depth study.

Many research results have also appeared for filters. Reference[12] explored the problem of dissipative asynchronous filtering for a class of T-S fuzzy MJSs in the continuous time domain. Reference[13] explored the design of robust fault detection

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for networked control systems with large transfer delays, and designed H_∞ fault detection filters based on the model developed. Although scholars have conducted extensive research on filters, there has been little study on the filtering domain of 2D MJSs based on deception attacks.

Therefore, inspired by the above discussion, In this article, we study and design an H_∞ filtration problems for 2D MJSs based on deception attacks. For the first time, deception attacks have been combined with 2D MJSs to study their H_∞ filtering problems, discussing the finite-region boundedness problem of 2D systems, and by solving them, the filter gains are obtained to prove the system stability of the system after being subjected to deception attacks. In summary, it is relevant to study the finite-region filtering problem for 2D MJSs under deception attacks. This paper contributes the following: 1.The studied system model incorporates more real-world factors, including 2D characteristics, Markov jump parameters, disturbances, and network attacks. 2.By employing suitable Lyapunov functions and specific recursive formulas, we successfully investigated the boundedness of the filtering error system within a finite region and derived the requisite conditions.

2 Problem Formulation

Take into account the following 2D MJSs constructed using the Roesser model:

$$\begin{cases} x^{\dagger}_{i,j} = A_{\alpha_{i,j}} x_{i,j} + B_{\alpha_{i,j}} \omega_{i,j}, \\ y_{i,j} = C_{\alpha_{i,j}} x_{i,j} + D_{\alpha_{i,j}} \omega_{i,j}, \\ z_{i,j} = L_{\alpha_{i,j}} x_{i,j}, \end{cases} \tag{1}$$

where $x^{\dagger}_{i,j} = col\begin{bmatrix} x^h_{i+1,j} & x^v_{i,j+1} \end{bmatrix}$, $x_{i,j} = col\begin{bmatrix} x^h_{i,j} & x^v_{i,j} \end{bmatrix}$, $x_{i,j} \in \mathrm{R}^n$ is the system state, including horizontal state $x^h_{i,j}$ and vertical state $x^v_{i,j}$. $y_{i,j} \in \mathrm{R}^y$ and $\omega_{i,j} \in \mathrm{R}^\omega$ represent the measured output and the external disturbance, where $\omega_{i,j}$ is subject to $\sum_{i,j\in \mathrm{N}_1\times N_2} \omega^{\mathrm{T}}_{i,j}\omega_{i,j} < d$. $z_{i,j} \in \mathrm{R}^z$ is the signal to estimate. $A_{\alpha_{i,j}}, B_{\alpha_{i,j}}, C_{\alpha_{i,j}}, D_{\alpha_{i,j}}, D_{\alpha_{i,j}}, L_{\alpha_{i,j}}$ are the known system matrices, which rely on the Markov chain $\alpha_{i,j}$. The values of $\alpha_{i,j}$ shift in $\mathcal{M}_1 = \{1, 2, \ldots, H_1\}$ with probability matrix $\Pi = \{\pi_{ij}\}$:

$$\mathrm{Prob}\{\alpha_{i+1,j} = n | \alpha_{i,j} = m\} = \mathrm{Prob}\{\alpha_{i,j+1} = n | \alpha_{i,j} = m\} = \pi_{mn}. \tag{2}$$

where $\pi_{ij} \in [0, 1]$ and $\sum_{j=1}^{\alpha_0} \pi_{ij} = 1$ for $\forall i, j \in \mathcal{M}_1$.

we designed the filter as follows:

$$\begin{cases} \hat{x}^{\dagger}_{i,j} = A_{f\alpha_{i,j}} \hat{x}_{i,j} + B_{f\alpha_{i,j}} \tilde{y}_{i,j}, \\ \hat{z}_{i,j} = L_{f\alpha_{i,j}} \hat{x}_{i,j}, \end{cases} \tag{3}$$

where $\hat{x}^{\dagger}_{i,j} = col\begin{bmatrix} \hat{x}^h_{i+1,j} & \hat{x}^v_{i,j+1} \end{bmatrix}, \hat{x}_{i,j} = col\begin{bmatrix} \hat{x}^h_{i,j} & \hat{x}^v_{i,j} \end{bmatrix}$, $\hat{x}_{i,j} \in \mathrm{R}^n$ is the filter state, including horizontal state $\hat{x}^h_{i,j}$ and vertical state $\hat{x}^v_{i,j}$. $\hat{z}_{i,j} \in \mathrm{R}^z$ is the filter output. $A_{f\alpha_{i,j}}, B_{f\alpha_{i,j}}, L_{f\alpha_{i,j}}$ are the filter parameters.

This paper focuses on deception attacks, where attackers may inject malicious information into the measurement signals. Accordingly, the actual input to the filter can be described as follows:

$$\tilde{y}_{i,j} = y_{i,j} + \xi_{i,j}(-y_{i,j} + \delta_{i,j}) \tag{4}$$

where $\delta_{i,j}$ represents deception attacks signal and $||\delta_{i,j}||_2 \leq ||Fy_{i,j}||_2$, $F > 0$ is a constant matrix used to denote the upper bound of the deception attacks function; $\xi_{i,j}$ denotes a stochastic variable that indicates the presence of a deception attack. This variable adheres to the conditions of a Bernoulli stochastic binary distribution:

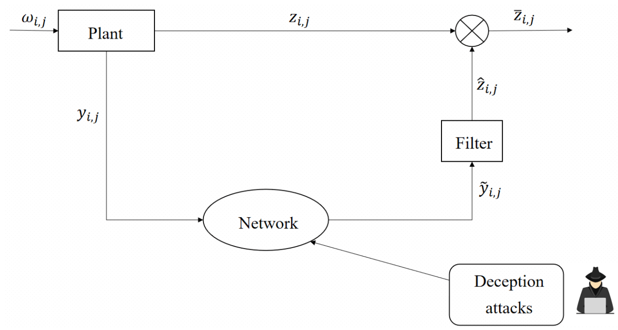

Figure 1: The framework of 2D MJSs with deception attacks

$$\mathrm{Prob}\{\xi_{i,j} = 1\} = \xi_1, \mathrm{Prob}\{\xi_{i,j} = 0\} = \bar{\xi}_1 = 1 - \xi_1 \tag{5}$$

where $\xi_1 \in [0, 1]$, ξ_1 is a known constant.

Considering (1) , (4) and (3) with $\alpha_{i,j} = m$. From this, we can formulate the following filtering error system:

$$\begin{cases} \bar{x}^{\dagger}_{i,j} = & \bar{A}_m \bar{x}_{i,j} + \bar{C}_m \omega_{i,j} + \bar{G}_m \delta_{i,j} + (\xi_{i,j} - \xi_1) \times [\bar{B}_m \bar{x}_{i,j} + \bar{D}_m \omega_{i,j} + \bar{H}_m \delta_{i,j}], \\ \bar{z}_{i,j} = & \bar{L}_m \bar{x}_{i,j}, \end{cases} \tag{6}$$

where $\bar{x}^{\dagger}_{i,j} = col\begin{bmatrix} \bar{x}^h_{i+1,j} & \bar{x}^v_{i,j+1} \end{bmatrix}, \bar{x}_{i,j} = col\begin{bmatrix} \bar{x}^h_{i,j} & \bar{x}^v_{i,j} \end{bmatrix}, \bar{z}_{i,j} = z_{i,j} - \hat{z}_{i,j}, \bar{x}^h_{i,j} = col\begin{bmatrix} x^h_{i,j} & x^h_{i,j} - \hat{x}^h_{i,j} \end{bmatrix}$, $\bar{x}^v_{i,j} = col\begin{bmatrix} x^v_{i,j} & x^v_{i,j} - \hat{x}^v_{i,j} \end{bmatrix}, \bar{A}_m = \begin{bmatrix} \bar{A}_{11m} & \bar{A}_{12m} \\ \bar{A}_{21m} & \bar{A}_{22m} \end{bmatrix}, \bar{A}_{11m} = \begin{bmatrix} A_{11m} & 0 \\ A_{11m} - (1-\xi_1)B_{f1m}C_{1m} - A_{f11m} & A_{f11m} \end{bmatrix}$, $\bar{A}_{12m} = \begin{bmatrix} A_{12m} & 0 \\ A_{12m} - (1-\xi_1)B_{f1m}C_{2m} - A_{f12m} & A_{f12m} \end{bmatrix}, \bar{A}_{21m} = \begin{bmatrix} A_{21m} & 0 \\ A_{21m} - (1-\xi_1)B_{f2m}C_{1m} - A_{f21m} & A_{f21m} \end{bmatrix}$ $\bar{A}_{22m} = \begin{bmatrix} A_{22m} & 0 \\ A_{22m} - (1-\xi_1)B_{f2m}C_{2m} - A_{f22m} & A_{f22m} \end{bmatrix}, \bar{B}_m = \begin{bmatrix} \bar{B}_{11m} & \bar{B}_{12m} \\ \bar{B}_{21m} & \bar{B}_{22m} \end{bmatrix}, \bar{B}_{11m} = \begin{bmatrix} 0 & 0 \\ B_{f1m}C_{1m} & 0 \end{bmatrix}$, $\bar{B}_{12m} = \begin{bmatrix} 0 & 0 \\ B_{f1m}C_{2m} & 0 \end{bmatrix}, \bar{B}_{21m} = \begin{bmatrix} 0 & 0 \\ B_{f2m}C_{1m} & 0 \end{bmatrix}, \bar{B}_{22m} = \begin{bmatrix} 0 & 0 \\ B_{f2m}C_{2m} & 0 \end{bmatrix}, \bar{C}_m = \begin{bmatrix} \bar{C}_{1m} \\ \bar{C}_{2m} \end{bmatrix}$, $\bar{C}_{1m} = \begin{bmatrix} B_{1m} \\ B_{1m} - (1-\xi_1)B_{f1m}D_m \end{bmatrix}, \bar{C}_{2m} = \begin{bmatrix} B_{2m} \\ B_{2m} - (1-\xi_1)B_{f2m}D_m \end{bmatrix}, \bar{D}_m = \begin{bmatrix} \bar{D}_{1m} \\ \bar{D}_{2m} \end{bmatrix}, \bar{D}_{1m} = \begin{bmatrix} 0 \\ B_{f1m}D_m \end{bmatrix}$, $\bar{D}_{2m} = \begin{bmatrix} 0 \\ B_{f2m}D_m \end{bmatrix}, \bar{G}_m = \begin{bmatrix} \bar{G}_{1m} \\ \bar{G}_{2m} \end{bmatrix}, \bar{G}_{1m} = \begin{bmatrix} 0 \\ -B_{f1m}\xi_1 \end{bmatrix}, \bar{G}_{2m} = \begin{bmatrix} 0 \\ -B_{f2m}\xi_1 \end{bmatrix}, \bar{H}_m = col\begin{bmatrix} \bar{\mathrm{H}}_{1m} & \bar{H}_{2m} \end{bmatrix}$,

$\bar{H}_{1m} = col\begin{bmatrix} 0 & -B_{f1m}\end{bmatrix}, \bar{H}_{2m} = col\begin{bmatrix} 0 & -B_{f2m}\end{bmatrix}, \bar{L}_m = \begin{bmatrix} \bar{L}_{1m} & \bar{L}_{2m}\end{bmatrix}, \bar{L}_{1m} = \begin{bmatrix} L_{1m} - L_{f1m} & L_{f1m}\end{bmatrix}$,
$\bar{L}_{2m} = \begin{bmatrix} L_{2m} - L_{f2m} & L_{f2m}\end{bmatrix}, \mathbf{E}\{\xi_{i,j} - \xi_1\} = 0, \mathbf{E}\{(\xi_{i,j} - \xi_1)^2\} = \xi_1\bar{\xi}_1$,

Definition 2.1. [14] Given positive scalars $c, c_1, c_2, N_1, N_2 \in \mathcal{N}_+$ and weighting matrices $R_h > 0, R_v > 0$, for $\forall\,(i,j) \in N_1 \times N_2$, the filtering error system (6) is finite-region bounded, such that

$$\begin{cases} \bar{x}_{0,j}^{h^{\mathrm{T}}}\tilde{R}^h\bar{x}_{0,j}^h \le c_1, \\ \bar{x}_{i,0}^{v^{\mathrm{T}}}\tilde{R}^v\bar{x}_{i,0}^v \le c_2, \end{cases} \Rightarrow \mathbf{E}\left\{\bar{x}_{i,j}^{\mathrm{T}}\begin{bmatrix}\tilde{R}^h & 0\\ 0 & \tilde{R}^v\end{bmatrix}\bar{x}_{i,j}\right\} < c, \tag{7}$$

where $\tilde{R}^h = diag\{R^h, R^h\}$, $\tilde{R}^v = diag\{R^v, R^v\}$

Definition 2.2. [14] The filtering error system (6) performs the H_∞ disturbance attenuation performance with $\gamma > 0$, such that inequality (7) and the following inequality:

$$\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\mathbf{E}\{|\bar{z}_{i,j}|^2\} \le \gamma^2\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}|\omega_{i,j}|^2. \tag{8}$$

3 Main results

Theorem 3.1. *For given positive scalars $c, c_1, c_2, d > 0, N_1, N_2 \in \mathcal{N}_+$, and weight matrices $R^h > 0$, $R^v > 0$, $\forall\,(i,j) \in N_1 \times N_2$, if there exist scalars α, γ, ξ_1, the filtering error system (6) is finite-region bounded.It satisfies the H_∞ disturbance weakening performance (8), such that:*

$$\begin{bmatrix} \mathcal{P}_m & \Psi_{1m}^{\mathrm{T}} & \Psi_{2m}^{\mathrm{T}} & \Psi_{3m}^{\mathrm{T}} \\ * & \mathcal{P}_n & 0 & 0 \\ * & * & -\xi_1 I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{9}$$

$$\frac{\alpha_0(N_1+N_2)\left\{\left[\lambda_{\max}(\bar{P}_m^h)c_1 + \lambda_{\max}(\bar{P}_m^v)c_2\right] + \gamma^2 d\right\}}{\min\left\{\lambda_{\min}(\bar{P}_m^h), \lambda_{\min}(\bar{P}_m^h)\right\}} < c \tag{10}$$

where $\mathcal{P}_m = -diag\{\alpha\tilde{P}_m^h, \alpha\tilde{P}_m^v, \tilde{P}_m^h, \tilde{P}_m^v, \gamma^2 I, \xi_1 I\}$, $\Psi_{1m}^{\mathrm{T}} = \left[\sqrt{\pi_{m1}}\Psi_{11}^{\mathrm{T}'} \quad \sqrt{\pi_{m2}}\Psi_{12}^{\mathrm{T}'} \cdots \sqrt{\pi_{mH_1}}\Psi_{1H_1}^{\mathrm{T}'}\right]$,
$\Psi_{1m}^{\mathrm{T}'} = \left[\Theta_{1m} \quad \Theta_{2m} \quad \Theta_{1m} \quad \Theta_{2m} \quad \Theta_{3m} \quad \Theta_{4m}\right]^{\mathrm{T}}$, $\Theta_{1m} = \left[\bar{A}_{11m}^{\mathrm{T}} \quad \sqrt{\xi_1\bar{\xi}_1}\bar{B}_{11m}^{\mathrm{T}} \quad \bar{A}_{21m}^{\mathrm{T}} \quad \sqrt{\xi_1\bar{\xi}_1}\bar{B}_{12m}^{\mathrm{T}}\right]^{\mathrm{T}}$,
$\Theta_{2m} = \left[\bar{A}_{12m}^{\mathrm{T}} \quad \sqrt{\xi_1\bar{\xi}_1}\bar{B}_{12m}^{\mathrm{T}} \quad \bar{A}_{22m}^{\mathrm{T}} \quad \sqrt{\xi_1\bar{\xi}_1}\bar{B}_{22m}^{\mathrm{T}}\right]^{\mathrm{T}}$, $\Theta_{3m} = \left[\bar{C}_{1m}^{\mathrm{T}} \quad \sqrt{\xi_1\bar{\xi}_1}\bar{D}_{1m}^{\mathrm{T}} \quad \bar{C}_{2m}^{\mathrm{T}} \quad \sqrt{\xi_1\bar{\xi}_1}\bar{D}_{2m}^{\mathrm{T}}\right]^{\mathrm{T}}$,
$\Theta_{4m} = \left[\bar{G}_{1m}^{\mathrm{T}} \quad \sqrt{\xi_1\bar{\xi}_1}\bar{H}_{1m}^{\mathrm{T}} \quad \bar{G}_{2m}^{\mathrm{T}} \quad \sqrt{\xi_1\bar{\xi}_1}\bar{H}_{2m}^{\mathrm{T}}\right]^{\mathrm{T}}$, $\Psi_{2m}^{\mathrm{T}} = \left[FC_{1m}H \quad FC_{2m}H \quad FC_{1m}H \quad FC_{2m}H \quad FD_m\right]^{\mathrm{T}}$, $H = [I \quad 0]$,
$\Psi_{3m}^{\mathrm{T}} = \left[\bar{L}_{1m} \quad \bar{L}_{2m} \quad \bar{L}_{1m} \quad \bar{L}_{2m} \quad 0 \quad 0\right]^{\mathrm{T}}$, $\mathcal{P}_n = -diag\{\underbrace{P_n^{h^{-1}}\ldots P_n^{h^{-1}}}_{2H_1}, \underbrace{P_n^{h^{-1}}\ldots P_n^{h^{-1}}}_{2H_1}, \underbrace{P_n^{v^{-1}}\ldots P_n^{v^{-1}}}_{2H_1}, \underbrace{P_n^{v^{-1}}\ldots P_n^{v^{-1}}}_{2H_1}\}$,
$\bar{P}_m^h = R^{h^{-\frac{1}{2}}}P_m^h R^{h^{-\frac{1}{2}}}$, $\bar{P}_m^v = R^{v^{-\frac{1}{2}}}P_m^v R^{v^{-\frac{1}{2}}}$, $\tilde{P}_m^h = diag\{P_m^h, P_m^h\}$, $\tilde{P_m^h} = diag\{P_m^v, P_m^v\}$,

Proof: To define the Lyapunov functions as follows:

$$V(i,j) = V_1(i,j) + V_2(i,j) = \bar{x}_{i,j}^{h^{\mathrm{T}}}P_m^h\bar{x}_{i,j}^h + \bar{x}_{i,j}^{v^{\mathrm{T}}}P_m^v\bar{x}_{i,j}^v \tag{11}$$

$$\Pi_{y,\delta} = \xi_1 y_{i,j}^{\mathrm{T}}F^{\mathrm{T}}Fy_{i,j} - \xi\delta_{i,j}^{\mathrm{T}}\delta_{i,j} \ge 0 \tag{12}$$

By means of the Schur complement, we can infer from (9) that

$$\mathbf{E}\left\{V_1(i+1,j) + V_2(i,j+1) - \alpha V(i,j) - \gamma^2\omega_{i,j}^{\mathrm{T}}\omega_{i,j}\right\} + \Pi_{y,\delta} < 0 \tag{13}$$

Combining (12) and (13), we have

$$\mathbf{E}\left\{V_1(i+1,j) + V_2(i,j+1) - \alpha V(i,j) - \gamma^2\omega_{i,j}^{\mathrm{T}}\omega_{i,j}\right\} < 0 \tag{14}$$

let $\tilde{V}(k) = \sum_{i+j=k}V(i,j)$, $\tilde{W}(k) = \sum_{i+j=k}\omega_{i,j}^{\mathrm{T}}\omega_{i,j}$, for $k < N_1 + N_2$,

$$\mathbf{E}\left\{\tilde{V}(k) - \alpha\tilde{V}(k-1)\right\} < \mathbf{E}\left\{V_1(0,k) + V_2(k,0) + \gamma^2\tilde{W}(k-1)\right\} \tag{15}$$

By applying successive substitutions of (15) and since $\tilde{W}(1) \ge 0$, we obtain

$$\begin{aligned} \mathbf{E}\left\{\tilde{V}(k) - \alpha^{k-1}\tilde{V}(1)\right\} &= \mathbf{E}\big\{\tilde{V}(k) - \alpha\tilde{V}(k-1) + \alpha\left[\tilde{V}(k-1) - \alpha\tilde{V}(k-2)\right] \\ &\quad + \cdots + \alpha^{k-2}\left[\tilde{V}(3) - \alpha\tilde{V}(1)\right]\big\} \\ &< \mathbf{E}\big\{V_1(0,k) + V_2(k,0) + \alpha\left[V_1(0,k-1) + V_2(k-1,0)\right] \\ &\quad + \cdots + \alpha^{k-2}\left[V_1(0,2) + V_2(2,0)\right] \\ &\quad + \gamma^2\tilde{W}(k-1) + \alpha\gamma^2\tilde{W}(k-2) + \cdots + \alpha^{k-1}\gamma^2\tilde{W}(1)\big\} \\ &= \mathbf{E}\big\{\sum_{l=2}^{k}\alpha^{k-l}\left[V_1(0,l) + V_2(l,0) + \gamma^2\tilde{W}(l)\right]\big\} \end{aligned} \tag{16}$$

$$\begin{aligned} \mathbf{E}\left\{\tilde{V}(k)\right\} &< \mathbf{E}\left\{\sum_{l=1}^{k}\alpha^{k-l}\left[V_1(0,l) + V_2(l,0) + \gamma^2\tilde{W}(l)\right]\right\} \\ &\le \max\{1, \alpha^{N_1+N_2}\}(N_1+N_2)\left[\lambda_{\max}(\bar{P}_m^h)c_1 + \lambda_{\max}(\bar{P}_m^v)c_2 + \gamma^2 d\right] \end{aligned} \tag{17}$$

on the other hand,

$$\begin{aligned} \mathbf{E}\left\{\tilde{V}(k)\right\} &\ge \mathbf{E}\left\{\bar{x}_{i,j}^{h^{\mathrm{T}}}P_m^h\bar{x}_{i,j}^h + \bar{x}_{i,j}^{v^{\mathrm{T}}}P_m^v\bar{x}_{i,j}^v\right\} \ge \lambda_{\min}(\bar{P}_m^h)\bar{x}_{i,j}^{h^{\mathrm{T}}}R^h\bar{x}_{i,j}^h + \lambda_{\min}(\bar{P}_m^v)\bar{x}_{i,j}^{v^{\mathrm{T}}}R^v\bar{x}_{i,j}^v \\ &\ge \min\left\{\lambda_{\min}(\bar{P}_m^h), \lambda_{\min}(\bar{P}_m^v)\right\}\bar{x}_{i,j}^{\mathrm{T}}R\bar{x}_{i,j} \end{aligned} \tag{18}$$

Putting together (17) with (18),we have

$$\bar{x}_{i,j}^{\mathrm{T}}R\bar{x}_{i,j} < \frac{\max\{1, \alpha^{N_1+N_2}\}(N_1+N_2)\left[\lambda_{\max}(\bar{P}_m^h)c_1 + \lambda_{\max}(\bar{P}_m^v)c_2 + \gamma^2 d\right]}{\min\left\{\lambda_{\min}(\bar{P}_m^v), \lambda_{\min}(\bar{P}_m^v)\right\}} < c \tag{19}$$

Consequently, the filtering error system (6) is finite-region bounded .This is the end of the proof of the finite-region bounded part.

By Schur complement, we are able to deduce from (9) that

$$\mathbf{E}\left\{V_1(i+1,j) + V_2(i,j+1) - V(i,j) - \gamma^2\omega_{i,j}^{\mathrm{T}}\omega_{i,j} + \bar{z}_{i,j}^{\mathrm{T}}\bar{z}_{i,j}\right\} + \Pi_{y,\delta} < 0 \tag{20}$$

Combining (12) and (20), we get

$$\mathbf{E}\left\{V_1(i+1,j) + V_2(i,j+1) - V(i,j) - \gamma^2\omega_{i,j}^{\mathrm{T}}\omega_{i,j} + \bar{z}_{i,j}^{\mathrm{T}}\bar{z}_{i,j}\right\} < 0 \tag{21}$$

Summing both side from 0 to N_1 and 0 to N_2, we have

$$\sum_{j=0}^{N_2}\left[\bar{x}^{h^{\mathrm{T}}}_{N_1+1,j}\tilde{P}^h_{\alpha_{N_1+1,j}}-\bar{x}^{h^{\mathrm{T}}}_{0,j}\tilde{P}^h_{\alpha_{0,j}}\bar{x}^h_{0,j}\right]+\sum_{i=0}^{N_1}\left[\bar{x}^{v^{\mathrm{T}}}_{i,N_2+1}\tilde{P}^v_{\alpha_i,N_2+1}\bar{x}^v_{i,N_2+1}-\bar{x}^{v^{\mathrm{T}}}_{i,0}\tilde{P}^v_{\alpha_{i,0}}\bar{x}^v_{i,0}\right]<\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\mathbf{E}\left\{\gamma^2\omega^{\mathrm{T}}_{i,j}\omega_{i,j}-\bar{z}^{\mathrm{T}}_{i,j}\bar{z}_{i,j}\right\} \tag{22}$$

By considering the zero bound conditions, we get:

$$\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\mathbf{E}\left\{\gamma^2\omega^{\mathrm{T}}_{i,j}\omega_{i,j}-\bar{z}^{\mathrm{T}}_{i,j}\bar{z}_{i,j}\right\}>0 \tag{23}$$

$$\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}\mathbf{E}\left\{|\bar{z}_{i,j}|^2\right\}<\gamma^2\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}|\omega_{i,j}|^2 \tag{24}$$

Thus, the filtering error system (6) is finite-region bounded .It satisfies the finite-region H_∞ performance condition. The proof is finished.

Theorem 3.2. *The filtering error system (6) is finite-region bounded . It satisfies the H_∞ capabilities decay level (8), for $m\in H_1$, if there exist symmetric matrices P^h_m, P^v_m, positive definite symmetric matrices G_1, G_2, scalars $\gamma>0, \psi_1>0, \psi_2>0, \psi_3>0, \psi_4>0$ and matrices $\hat{A}_{f11m}, \hat{A}_{f12m}, \hat{A}_{f21m}, \hat{A}_{f22m}, \hat{B}_{f1m}, \hat{B}_{f2m}, \hat{L}_{f1m}, \hat{L}_{f2m}$, such that*

$$\begin{bmatrix} \mathcal{P}'_n & 0 & 0 & \Psi_{1m} \\ * & -\xi_1 I & 0 & \Psi_{2m} \\ * & * & -I & \Psi_{3m} \\ * & * & * & \mathcal{P}_m \end{bmatrix}<0 \tag{25}$$

$$\psi_1<P^h_m<\psi_2 \tag{26}$$

$$\psi_3<P^v_m<\psi_4 \tag{27}$$

$$\begin{bmatrix} -c+\frac{\alpha^0(N_1+N_2)d}{\psi_0} & \sqrt{\alpha^0(N_1+N_2)(\psi_2 c_1+\psi_4 c_2)} \\ * & -\psi_0 \end{bmatrix} \tag{28}$$

where

$$\mathcal{P}'_n=*-diag\{\underbrace{(G_1^{\mathrm{T}}-P^h_n+G_1)\dots(G_1^{\mathrm{T}}-P^h_n+G_1)}_{4H_1},\underbrace{(G_2^{\mathrm{T}}-P^v_n+G_2)\dots(G_2^{\mathrm{T}}-P^v_n+G_2)}_{4H_1}\},$$

$\Psi_{1m}=\left[\sqrt{\pi_{m1}}\Psi^{\mathrm{T}'}_{11}\quad\sqrt{\pi_{m2}}\Psi^{\mathrm{T}'}_{12}\dots\sqrt{\pi_{mH_1}}\Psi^{\mathrm{T}'}_{1H_1}\right]^{\mathrm{T}}, \Psi'_{1m}=\left[\Theta'_{1m}\quad\Theta'_{2m}\quad\Theta'_{1m}\quad\Theta'_{2m}\quad\Theta'_{3m}\quad\Theta'_{4m}\right]$,
$\Theta'_{1m}=\left[\bar{A}^{\mathrm{T}'}_{11m}\quad\sqrt{\xi_1\bar{\xi}_1}\bar{B}^{\mathrm{T}'}_{11m}\quad\bar{A}^{\mathrm{T}'}_{21m}\quad\sqrt{\xi_1\bar{\xi}_1}\bar{B}^{\mathrm{T}'}_{12m}\right]^{\mathrm{T}}, \Theta_{2m}=\left[\bar{A}^{\mathrm{T}'}_{12m}\quad\sqrt{\xi_1\bar{\xi}_1}\bar{B}^{\mathrm{T}'}_{12m}\quad\bar{A}^{\mathrm{T}'}_{22m}\quad\sqrt{\xi_1\bar{\xi}_1}\bar{B}^{\mathrm{T}'}_{22m}\right]^{\mathrm{T}}$,
$\Theta_{3m}=\left[\bar{C}^{\mathrm{T}'}_{1m}\quad\sqrt{\xi_1\bar{\xi}_1}\bar{D}^{\mathrm{T}'}_{1m}\quad\bar{C}^{\mathrm{T}'}_{2m}\quad\sqrt{\xi_1\bar{\xi}_1}\bar{D}^{\mathrm{T}'}_{2m}\right]^{\mathrm{T}}, \Theta_{4m}=\left[\bar{G}^{\mathrm{T}'}_{1m}\quad\sqrt{\xi_1\bar{\xi}_1}\bar{H}^{\mathrm{T}'}_{1m}\quad\bar{G}^{\mathrm{T}'}_{2m}\quad\sqrt{\xi_1\bar{\xi}_1}\bar{H}^{\mathrm{T}'}_{2m}\right]^{\mathrm{T}}$,
$\bar{A}'_{11m}=\begin{bmatrix} G_1^{\mathrm{T}}A_{11m} & 0 \\ G_1^{\mathrm{T}}A_{11m}-(1-\xi_1)\hat{B}_{f1m}C_{1m}-\hat{A}_{f11m} & \hat{A}_{f11m}\end{bmatrix}, \bar{B}'_{11m}=\begin{bmatrix} 0 & 0 \\ \hat{B}_{f1m}C_{1m} & 0\end{bmatrix}, \bar{D}_{1m}=\begin{bmatrix} 0 \\ \hat{B}_{f1m}D_m\end{bmatrix}$,
$\bar{A}'_{12m}=\begin{bmatrix} G_1^{\mathrm{T}}A_{12m} & 0 \\ G_1^{\mathrm{T}}A_{12m}-(1-\xi_1)\hat{B}_{f1m}C_{2m}-\hat{A}_{f12m} & \hat{A}_{f12m}\end{bmatrix}, \bar{B}'_{12m}=\begin{bmatrix} 0 & 0 \\ \hat{B}_{f1m}C_{2m} & 0\end{bmatrix}, \bar{D}_{2m}=\begin{bmatrix} 0 \\ \hat{B}_{f2m}D_m\end{bmatrix}$,
$\bar{A}'_{21m}=\begin{bmatrix} G_2^{\mathrm{T}}A_{21m} & 0 \\ G_2^{\mathrm{T}}A_{21m}-(1-\xi_1)\hat{B}_{f2m}C_{1m}-\hat{A}_{f21m} & \hat{A}_{f21m}\end{bmatrix}, \bar{B}'_{21m}=\begin{bmatrix} 0 & 0 \\ \hat{B}_{f2m}C_{1m} & 0\end{bmatrix}, \bar{G}_{1m}=\begin{bmatrix} 0 \\ -\hat{B}_{f1m}\xi_1\end{bmatrix}$,
$\bar{A}'_{22m}=\begin{bmatrix} G_2^{\mathrm{T}}A_{22m} & 0 \\ G_2^{\mathrm{T}}A_{22m}-(1-\xi_1)\hat{B}_{f2m}C_{2m}-\hat{A}_{f22m} & \hat{A}_{f22m}\end{bmatrix}, \bar{B}'_{22m}=\begin{bmatrix} 0 & 0 \\ \hat{B}_{f2m}C_{2m} & 0\end{bmatrix}, \bar{G}_{2m}=\begin{bmatrix} 0 \\ -\hat{B}_{f2m}\xi_1\end{bmatrix}$,
$\bar{C}'_{1m}=\begin{bmatrix} G_1^{\mathrm{T}}B_{1m} \\ G_1^{\mathrm{T}}B_{1m}-(1-\xi_1)\hat{B}_{f1m}D_m\end{bmatrix}, \bar{C}'_{2m}=\begin{bmatrix} G_2^{\mathrm{T}}B_{2m} \\ G_2^{\mathrm{T}}B_{2m}-(1-\xi_1)\hat{B}_{f2m}D_m\end{bmatrix}, \bar{H}_{1m}=col\begin{bmatrix} 0 & -\hat{B}_{f1m}\end{bmatrix}$,
$\bar{H}_{2m}=col\begin{bmatrix} 0 & -\hat{B}_{f2m}\end{bmatrix}, \psi_0=\min\left\{\lambda_{\min}(\bar{P}^h_m), \lambda_{\min}(\bar{P}^h_m)\right\}$. *The filter gains are given by* $A_{f11m}=G_1^{-\mathrm{T}}\hat{A}_{f11m}$, $A_{f12m}=G_1^{-\mathrm{T}}\hat{A}_{f12m}, A_{f21m}=G_2^{-\mathrm{T}}\hat{A}_{f21m}, A_{f22m}=G_2^{-\mathrm{T}}\hat{A}_{f22m}, B_{f1m}=G_1^{-\mathrm{T}}\hat{B}_{f1m}, B_{f2m}=G_2^{-\mathrm{T}}\hat{B}_{f2m}$, $L_{f1m}=\hat{L}_{f1m}, L_{f2m}=\hat{L}_{f2m}$, *such that:*

Proof: Using Schur compensation, we can obtain

$$\begin{bmatrix} \mathcal{P}_n & 0 & 0 & \Psi_{1m} \\ * & -\xi_1 I & 0 & \Psi_{2m} \\ * & * & -I & \Psi_{3m} \\ * & * & * & \mathcal{P}_m \end{bmatrix}<0 \tag{29}$$

Using the matrix $diag\{\underbrace{G_1^{\mathrm{T}}\dots G_1^{\mathrm{T}}}_{4H_1},\underbrace{G_2^{\mathrm{T}}\dots G_2^{\mathrm{T}}}_{4H_1},I,I,I,I,I,I,I,I\}$,, we perform an equivalence transformation on inequality (29),resulting in:

$$\begin{bmatrix} \hat{\mathcal{P}}_n & 0 & 0 & \Psi_{1m} \\ * & -\xi_1 I & 0 & \Psi_{2m} \\ * & * & -I & \Psi_{3m} \\ * & * & * & \mathcal{P}_m \end{bmatrix}<0 \tag{30}$$

where

$$\hat{\mathcal{P}}_n=-diag\{\underbrace{(G_1^{\mathrm{T}}P^{h^{-1}}_n G_1)\dots(G_1^{\mathrm{T}}P^{h^{-1}}_n G_1)}_{4H_1},\underbrace{(G_2^{\mathrm{T}}P^{v^{-1}}_n G_2)\dots(G_2^{\mathrm{T}}P^{v^{-1}}_n G_2)}_{4H_1}$$

Due to the fact that $-G_1^{\mathrm{T}}P^{h^{-1}}_n G_1<-G_1^{\mathrm{T}}+P^h_n-G_1$ and $-G_2^{\mathrm{T}}P^{v^{-1}}_n G_2<-G_2^{\mathrm{T}}+P^v_n-G_2$, (30) is equivalent to (25). Next, we apply some constraints on P^h_m, P^v_m and the inequality (10) is transformed into inequality (28). The proof is finished.

4 simulation example

Here, we will demonstrate the effectiveness of the designed filter by simulating the Darboux equation [15]. The Darboux equation can be transformed into a 2D MJS by a technique similar to that used in [15]. For this case, this model of 2D MJSs consists of two modes of operation and the respective system matrices are as follows.

Mode 1: $A_{(1)} = \begin{bmatrix} 0 & 1 \\ 0.45 & 0.05 \end{bmatrix}, B_{(1)} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, C_{(1)} = \begin{bmatrix} 0.5 & 0 \end{bmatrix}, D_{(1)} = 0.1, L_{(1)} = \begin{bmatrix} 0.5 & 0.1 \end{bmatrix}$.

Mode 2: $A_{(2)} = \begin{bmatrix} 0 & -0.3 \\ 0.2 & 0 \end{bmatrix}, B_{(2)} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}, C_{(2)} = \begin{bmatrix} 0.1 & 0.6 \end{bmatrix}, D_{(2)} = D_{(1)}, L_{(2)} = L_{(1)}$.

The other parameters are set as: $c_1 = c_2 = 0.3, \alpha = 1.01, \xi_1 = 0.3, F = 0.3, d = 0.01, \vartheta_1 = 1, \vartheta_2 = 4, \vartheta_3 = 1, \vartheta_4 = 4, N_1 = N_2 = 10, R^h = R^v = I, \Pi = \begin{bmatrix} 0.3 & 0.7 \\ 0.8 & 0.2 \end{bmatrix}$. By solving the matrix inefficiency in Theorem 2, we get

Mode 1: $A_{f(1)} = \begin{bmatrix} -0.0146 & 0.8925 \\ 0.7718 & 0.3275 \end{bmatrix}, B_{f(1)} = \begin{bmatrix} 0.2380 \\ 0.6251 \end{bmatrix}, L_{f(1)} = \begin{bmatrix} 0.4019 & 0.00008 \end{bmatrix}$.

Mode 2: $A_{f(2)} = \begin{bmatrix} -0.0136 & 0.9198 \\ 0.3796 & 0.7774 \end{bmatrix}, B_{f(2)} = \begin{bmatrix} 0.3030 \\ 0.3859 \end{bmatrix}, L_{f(2)} = \begin{bmatrix} 0.4628 & -0.00007 \end{bmatrix}$. with $c = 56.0573$ and the optimized H_∞ noise attenuation performance $\gamma = 3.7067$. For $0 \leq i \leq 5, 0 \leq j \leq 5$, let the boundary condition $x_{i,0} = 0.2, x_{0,j} = 0.3$, and disturbance input $\omega_{i,j} = 0.1e^{-0.01j}\cos(2i)$. Assume the function of deception attacks: $\delta_{i,j} = -tanh(0.3y_{i,j})$. Fig. 2 illustrates the schema transformations (1) for 2D MJSs. The designed filter's error response regarding $\bar{z}_{i,j}$ is shown in the Fig. 3. From the Fig. 3, it can be observed that the filtering error $\bar{z}_{i,j}$ tends towards zero and stabilizes within a certain range.

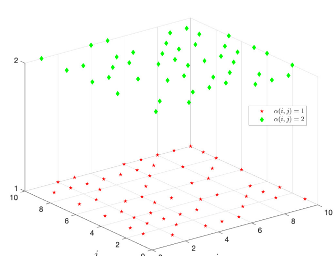

Figure 2: 2D MJSs modes

5 Conclusion

In this paper, we investigate a finite-region H_∞ filter design method for 2D MJSs with high transient performance requirements under random network attacks. The phenomenon of randomly occurring deception attacks is modeled in a unified way using Bernoulli processes to portray the phenomenon. Sufficient conditions are given to guarantee that the filtering error system is finite-region bounded

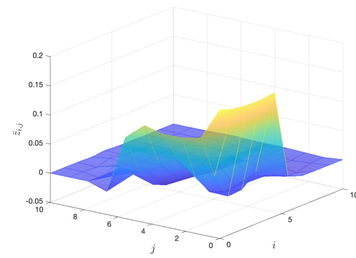

Figure 3: Filtering error $\bar{z}_{i,j}$

for a given level of H_∞ disturbance attenuation. Finally, numerical example is used to verify the effectiveness of the method proposed in this paper.

References

[1] Abhinav Kumar Singh, Ravindra Singh, and Bikash C Pal. Stability analysis of networked control in smart grids. *IEEE Transactions on Smart Grid*, 6(1):381–390, 2014.

[2] Dermot Putnam, Maria Kovacova, Katarina Valaskova, and Vojtech Stehel. The algorithmic governance of smart mobility: Regulatory mechanisms for driverless vehicle technologies and networked automated transport systems. *Contemporary Readings in Law and Social Justice*, 11(1):21–26, 2019.

[3] Behrooz Rahmani and Amir Hossein Davaie Markazi. Networked control of industrial automation systems—a new predictive method. *The International Journal of Advanced Manufacturing Technology*, 58:803–815, 2012.

[4] Woochul Kang, Lui Sha, Richard B Berlin, and Julian M Goldman. The design of safe networked supervisory medical systems using organ-centric hierarchical control architecture. *IEEE journal of biomedical and health informatics*, 19(3):1077–1086, 2014.

[5] Md Ali Reza Al Amin, Sachin Shetty, Laurent Njilla, Deepak K. Tosh, and Charles Kamhoua. Hidden markov model and cyber deception for the prevention of adversarial lateral movement. *IEEE Access*, 9:49662–49682, 2021.

[6] Derui Ding, Guoliang Wei, Sunjie Zhang, Yurong Liu, and Fuad E Alsaadi. On scheduling of deception attacks for discrete-time networked systems equipped with attack detectors. *Neurocomputing*, 219:99–106, 2017.

[7] Xiaobin Gao, Feiqi Deng, Hongyang Zhang, and Pengyu Zeng. Adaptive neural state estimation of markov jump systems under scheduling protocols and probabilistic deception attacks. *IEEE Transactions on Cybernetics*, 53(3):1830–1842, 2023.

[8] Wenhai Qi, Caiyu Lv, Ju H. Park, Guangdeng Zong, Jun Cheng, and Kaibo Shi. Smc for semi-markov jump cyber-physical systems subject to randomly occurring deception attacks. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 69(1):159–163, 2022.

[9] Peng Cheng, Shuping He, Xiaoli Luan, and Fei Liu. Finite-region asynchronous H_∞ control for 2d markov jump systems. *Automatica*, 129:109590, 2021.

[10] Ying Shen, Zheng-Guang Wu, Peng Shi, and Guanghui Wen. Dissipativity based fault detection for 2d markov jump systems with asynchronous modes. *Automatica*, 106:8–17, 2019.

[11] Xiaofeng Ma, Jingjing Dong, Weipeng Tai, Jianping Zhou, and Wojciech Paszke. Asynchronous event-triggered H_∞ control for 2d markov jump systems subject to networked random packet losses. *Communications in Nonlinear Science and Numerical Simulation*, 126:107453, 2023.

[12] Shanling Dong, Zheng-Guang Wu, Ya-Jun Pan, Hongye Su, and Yang Liu. Hidden-markov-model-based asynchronous filter design of nonlinear markov jump systems in continuous-time domain. *IEEE Transactions on Cybernetics*, 49(6):2294–2304, 2019.

[13] Zehui Mao, Bin Jiang, and Peng Shi. H_∞ fault detection filter design for networked control systems modelled by discrete markovian jump systems. *IET control theory & applications*, 1(5):1336–1343, 2007.

[14] Jiankang Fang, Chengcheng Ren, Hai Wang, Vladimir Stojanovic, and Shuping He. Finite-region asynchronous H_∞ filtering for 2-d markov jump systems in roesser model. *Applied Mathematics and Computation*, 470:128573, 2024.

[15] Ligang Wu, Peng Shi, Huijun Gao, and Changhong Wang. H_∞ filtering for 2d markovian jump systems. *Automatica*, 44(7):1849–1858, 2008.