Multiple-target enclosing control using bearing-only measurements

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***Abstract*—In the past few years, there has been significant progress in the development of enclosing control approaches for various applications in military and civilian domains. This study explores the use of a multi-agent system (MAS) equipped with bearing-only sensors to address the enclosing control problem for multiple targets in a constrained environment. The crux of this issue is to estimate the geometric center of multiple targets with limited information, such as bearing measurements to a subset of targets and message exchanges with neighboring agents. This paper proposes a composite estimator, by which an agent can firstly generate its individual *opinion* of the center by the collected bearing measurements to its observable targets; then their opinions are exchanged in the network to reach an agreement of the geometric center. Based on this, the MAS can achieve the target circular formation by rotating at a predetermined radius and rotational speed about the center. The proposed method's efficacy and efficiency are confirmed by thorough theoretical analysis and numerical simulation.**

***Index Terms*—Enclosing control, Multi-agent system, Bearing-only measurements, Geometric center.**

I. INTRODUCTION

The increasing incidence of the enclosing control problem relates to its broad applicability in diverse military and civilian fields, such as security, surveillance, and rescue operations [1]. Firstly, enclosing multiple targets would be more efficient but harder than the traditional single target case [2], [3]. Besides, a lot of enclosing missions will be tasked in confined environment, where the observation and communication is not as perfect as in a laboratory, and therefore the exact positions of targets can not be directly obtained in practice. This study addresses the enclosing control problem for multiple targets under restricted conditions, utilizing a MAS equipped solely with bearing sensors.

The enclosing problem for multiple targets is more challenging than the single cases. [4] deals with the finite-time navigation control problem for MAS and aims to accomplish the uniform distribution of agents around the designated target on a circle. [5] based on the encirclement control method for multi-agents surrounding multiple targets, develops a control algorithm that enables agents to collectively encircle multiple targets. [6] investigates the second-order multi-agent systems' closed-loop control problem with respect to static or moving targets. [7], [8] explores the collective encirclement problem of multiple targets by multi-agent systems and proposes a control algorithm to realize collective encirclement motion accordingly. In the absence of prior knowledge regarding the amplitude of the target speed, the enclosing control of the agent can also be realized [9]. In addition, the control input saturation and chattering phenomenon in the multi-target enclosing problem has been solved in [10] and [11] respectively.

However, in the domain of MAS, it is challenging for the agents to obtain complete information concerning the target's relative position. Thus, the absence or unavailability of comprehensive relative position data concerning the target poses a significant challenge for multi-agent systems' research, thereby restricting the scope of investigation. The prevalent measurement methods for target location information are distance measurement and bearing measurement. Compared with distance measurement, bearing measurement [3] only requires some common sensors, such as monocular camera system, to accomplish the measurement of target position information effectively. Moreover, mobile robots equipped with monocular camera systems are also easily accessible. Therefore, in recent years, an increasing number of researchers have shifted their focus to bearing measurement.

The author employs bearing measurement to locate and encircle a single target based on the agent's position and bearing [3], [12]. [13] considered the non-holonomic robots and developed target localization and encirclement algorithms using bearing measurement. They fulfilled the localization and encirclement of the targets through the deployment of single-agent systems. Encirclement control of targets by multiple agents using bearing measurements has also been investigated [14]. In other fields, such as unmanned surface ships, bearings-only control can also make surface ships surround moving targets [15], [16].

Encouraged by the aforementioned article, we investigate the challenge of enclosing multiple targets with the aid of bearing-only measurements. The primary contribution of this

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paper is to suggest a composite estimator, through which agents can first collect bearing measurements of their observable objects, and then the information they collect is exchanged in the network, and finally reach a consensus on the geometric center.

II. Preliminaries and Problem Description

A. Preliminaries

To begin with, the graph theory is important for modeling our problem. Let $\digamma = \{\Upsilon, \Lambda, A\}$ be an undirected connected graph, where $\Upsilon = \{\upsilon_1, \ldots, \upsilon_m\}$ denotes the node set with finite elements, $\Lambda(\digamma) = \{e_{kj} = (\upsilon_k, \upsilon_j)\} \subset \Upsilon \times \Upsilon$ represents the edge set, and $A = [a_{kj}] \in R^{m\times m}$ is the weighted adjacency matrix defined as

$$a_{kj} = \begin{cases} 1, & \text{if} \quad (\upsilon_k, \upsilon_j) \in \Lambda \\ 0, & otherwise \end{cases}$$

To ensure a single connected component in a graph for control science applications, it is essential that a path exists between any two nodes, which is both necessary and sufficient for the graph's connectivity. For a MAS composed of m targets and n agents, we denote N_i^T as the set of targets that can be observed by the i-th agent, and N_j^A as the set of agents that can communicate with agent j i.e. its neighboring set. We use the operator $|\cdot|$ to calculate the number of elements of a set e.g. $|N_j^A|$ is the number of neighbors of j-th agent, and $||.||$ stands for the Euclidean norm. The following assumption about the communication and observation topology is essential in our problem:

Assumption 1: Agents exhibit a constant and unidirectional communication topology. All targets are assumed to be universally observable by at least one agent in the decentralized control system.

The next lemma is presented to facilitate our following theoretical analysis.

Lemma 1: Let $\{\lambda_j(t)|j = 1, 2, ..., m\}and\{\tau_j(t)|j = 1, 2, ..., m\}$ be given sets of functions and $A = [a_{ji}] \in R^{m\times m}$ be a given matrix. If $a_{ji} = a_{ij}$, that is

$$\sum_{j=1}^m \sum_{i=1}^m [a_{ji}\lambda_j(t)\tfrac{\tau_i(t)-\tau_j(t)}{|\tau_i(t)-\tau_j(t|}]$$

$$=\tfrac{1}{2}\sum_{j=1}^m \sum_{i=1}^m \{a_{ji}[\lambda_j(t) - \lambda_i(t)]\tfrac{\tau_i(t)-\tau_j(t)}{|\tau_i(t)-\tau_j(t|}]\}.$$

It is worth noting that if $\lambda_j(t) = \tau_j(t)$, that is

$$\sum_{j=1}^m \sum_{i=1}^m [a_{ji}\tau_j(t)\tfrac{\tau_i(t)-\tau_j(t)}{|\tau_i(t)-\tau_j(t|}]$$

$$=-\tfrac{1}{2}\sum_{j=1}^m \sum_{i=1}^m [a_{ji}|\tau_i(t) - \tau_j(t)|].$$

In addition, if $\lambda_j(t) = 1, j = 1, 2, ..., m$, then it can be inferred that

$$\sum_{j=1}^m \sum_{i=1}^m [a_{ji}\tfrac{\tau_i(t)-\tau_j(t)}{|\tau_i(t)-\tau_j(t|}] = 0.$$

B. Problem description

We investigate a MAS composed of n agents and m targets in this paper. Graph $\digamma(t)$ shows the topological structure among multiple agents at time t. Assuming the i-th agent has the dynamics given by:

$$\dot{L}_{Ai}(t) = u_i(t), \tag{1}$$

where $L_{Ai}(t)$ represents the position of the i-th agent, and $u_i(t)$ represents the control input. The positions of m stationary targets are denoted by $L_{Tj}, \forall j \in \{1, 2, \cdots, m\}$. Opposite to the existing works [17], we assume that the agent is tasked in a confined environment. Thus they can only know their own exact position and observe a subset of targets by the equipped bearing-only sensors (e.g. camera or radar). The objective of this paper is to deploy those agents to enclose the targets by a predefined circular formation. An exemplary case of the problem with two agents and three targets is shown in Fig1. As depicted in the figure, the notations Γ_i is the relative bearing measurements and ψ_i the vector connecting the agent and the observed target.

Each agent formulates a bearing-based estimator for estimating the positions of multiple targets while computing their geometric centroid. Lastly, fixing the origin of circular formation at the center, the enclosing control law for the MAS can be designed by considering the expected radius r and rotating speed λ. The position estimation error for a stationary target is defined as:

$$\widetilde{L}_{Ti}(t) = \widehat{L}_{Ti}(t) - L_{Ti}(t), \tag{2}$$

where $\widehat{L}_{Ti}(t)$ is the estimated value of $L_{Ti}(t)$, $d_i(t) = ||L_{Ai}(t) - L_{Tj}(t)||$ denotes the true distance of the i-th agent to the observed j-th target, and $\hat{d}_i = ||L_{Ai}(t) - \hat{L}_{Tj}(t)||$ is the estimated distance.

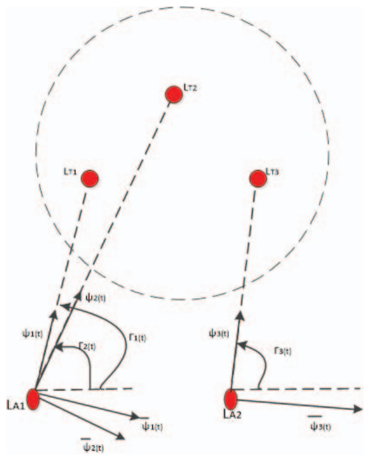

Fig. 1: Problem description and notations.

As stated earlier, the notation $\psi_i(t) = [cos(\Gamma_i(t)), sin(\Gamma_i(t))]^T$ denotes a unit vector that lies on the straight line connecting the target and the agent, which mathematically,

$$\psi_i(t) = \frac{L_{Tj}(t) - L_{Ai}(t)}{\|L_{Tj}(t) - L_{Ai}(t)\|} = \frac{L_{Tj}(t) - L_{Ai}(t)}{d_i(t)}, \quad (3)$$

and $\overline{\psi}_i(t) = [sin(\Gamma_i(t)), -cos(\Gamma_i(t))]^T \in R^2$ is the clockwise rotated unit vector that is perpendicular to $\psi_i(t)$.

III. Main Results

Comprehensive estimation and control are necessary for achieving circular motion around the target's geometric center at a predetermined radius. We introduce an algorithm to estimate both $L_{Tj}(t)$ and the target's geometric center, and examine the stability of the algorithm and control law.

Assume that the target is stationary ($\dot{L}_{Tj}(t) = 0$). Our first strategy is to develop a central estimator that avoids differentiation of measured data and achieves exponential convergence of $\widetilde{L}_{Tj}(t)$ to zero. Assuming κ_e is a regular number, the defined estimator for the target position is as follows:

$$\dot{\widehat{L}}_{Tj}(t) = \kappa_e(\varsigma - \psi_i(t)\psi_i^T(t))(L_{Ai}(t) - \widehat{L}_{Tj}(t)), \quad (4)$$

where ς denotes the identity matrix, and $\psi_i(t)\psi_i^T(t)$ represents the projection matrix on the vector $\psi_i(t)$. The estimation error dynamics for bearing measurement, derived by utilizing equations (2), (3) and (4), is as follows

$$\begin{aligned}\dot{\widetilde{L}}_{Tj}(t) &= \kappa_e(\varsigma - \psi_i(t)\psi_i^T(t))(L_{Ai}(t) - \widehat{L}_{Tj}(t)) \\ &= \kappa_e(\varsigma - \psi_i(t)\psi_i^T(t))(-\psi_i(t)d_i(t) - \widehat{L}_{Tj}(t) + L_{Tj}(t)) \\ &= -\kappa_e(\varsigma - \psi_i(t)\psi_i^T(t))\widetilde{L}_{Tj}(t) \\ &= -\kappa_e\overline{\psi}_i(t)\overline{\psi}_i^T(t)\widetilde{L}_{Tj}(t)\end{aligned} \quad (5)$$

To prove its stability, we firstly notice that that the eigenvalues of the symmetric matrix $\overline{\psi}_i(t)\overline{\psi}_i^T(t)$ is 0 and -1, which means it is negative semi definite. Therefore, regarding the error equation in (5), by choosing Lyapunov function $V = \frac{1}{2}\widetilde{L}_{Tj}^T\widetilde{L}_{Tj}$, we can see that $\dot{V} = -\kappa_e\left\|\overline{\psi}_i^T\widetilde{L}_{Tj}(t)\right\|^2$ is also negative semi-definite, which yields that equation (5) is uniformly stable. Therefore, $\widetilde{L}_{Tj}(t)$ and $\dot{\widehat{L}}_{Tj}(t)$ are all bounded.

In this paper, we assume that each target is observable by at least one agent and the agent can adapt its dynamic behavior based on bearing information. A bearing-based geometric center estimator is required for estimating the centers of multiple targets. The geometric center estimator based on bearing information is defined as follows:

$$\begin{aligned}\xi_i(t) &= \Phi_i(t) + \frac{n}{m}\sum_{j\in N_i^T}\frac{1}{|N_j^A|}\widehat{L}_{Tj}(t) \\ \dot{\Phi}_i(t) &= \kappa_c\sum_{j\in N_i} a_{ij}\frac{\xi_j(t) - \xi_i(t)}{|\xi_j(t) - \xi_i(t)|}\end{aligned} \quad (6)$$

where $\Phi_i(t)$ represents the internal state of the estimator and $\Phi_i(0) = 0$. $\xi_i(t) \in R^2$, which is the estimation of the geometric center of the target by agent i, Z^* ($Z^* = \frac{1}{m}\sum_{j=1}^m L_{Tj}(t)$) represents the centroids of multiple targets and κ_c denotes a regular number.

Theorem 3.1: All targets are assumed to be stationary. If $F(t)$ is connected, using algorithm (6) of system (1), we have$\lim_{t\to\infty}|\xi_j(t) - \xi_i(t)| = 0$, $\forall i = 1, ..., n$.

Proof: Theorem 3.1 is a special case of Theorem 2, and therefore, its proof is omitted.

Theorem 3.2: Assuming that all targets are dynamic. From equation (5), $\left\|\dot{\widehat{L}}_{Tj}(t)\right\| \leq \kappa_b$, If $F(t)$ is connected, and $\kappa_c \geq \kappa_b(n-1)$, using algorithm (6) for system (1), we get $\lim_{t\to\infty}|\xi_j(t) - \xi_i(t)| = 0$, ,$\forall i = 1, ...n$.

Proof: To prove the above theorem, define a Lyapunov function as:

$$V(t) = \frac{1}{2}\sum_{i=1}^n[\xi_i(t) - \frac{1}{n}\sum_{k=1}^n\xi_k(t)]^2 \quad (7)$$

According to equation(6), the derivative of $V(t)$ can be obtained

$$\begin{aligned}\dot{V}(t) = &\sum_{i=1}^n\{[\xi_i(t) - \frac{1}{n}\sum_{k=1}^n\xi_k(t)] \times [\dot{\Phi}_i(t) \\ &+ \frac{n}{m}\sum_{j\in N_i^T}\frac{1}{|N_j^A|}\dot{\widehat{L}}_{Tj}(t) - \frac{1}{n}\sum_{k=1}^n\dot{\xi}_k(t)]\}\end{aligned} \quad (8)$$

besides,

$$\begin{aligned}&\sum_{i=1}^n\{[\xi_i(t) - \frac{1}{n}\sum_{k=1}^n\xi_k(t)] \times [\dot{\Phi}_i(t)]\} \\ &= \kappa_c\sum_{i=1}^n\sum_{j\in N_i}[a_{ij}\xi_i(t)\frac{\xi_j(t) - \xi_i(t)}{|\xi_j(t) - \xi_i(t)|}] \\ &-\frac{\kappa_c}{n}\sum_{k=1}^n\xi_k(t)\sum_{i=1}^n\sum_{j\in N_i}[a_{ij}\frac{\xi_j(t) - \xi_i(t)}{|\xi_j(t) - \xi_i(t)|}]\end{aligned}$$

By utilizing Lemma 1, we achieve

$$\begin{aligned}&\sum_{i=1}^n\{[\xi_i(t) - \frac{1}{n}\sum_{k=1}^n\xi_k(t)] \times [\dot{\Phi}_i(t)]\} \\ &= -\frac{1}{2}\kappa_c\sum_{i=1}^n\sum_{j\in N_i}[a_{ij}|\xi_j(t) - \xi_i(t)|]\end{aligned} \quad (9)$$

Because $\left\|\dot{\widehat{L}}_{Tj}(t)\right\| \leq \kappa_b$, we can get

$$\begin{aligned}&\sum_{i=1}^n\{[\xi_i(t) - \frac{1}{n}\sum_{k=1}^n\xi_k(t)] \times \frac{n}{m}\sum_{j\in N_i^T}\frac{1}{|N_j^A|}\dot{\widehat{L}}_{Tj}(t)\} \\ &\leq \frac{\kappa_b(n-1)}{2}\sum_{i=1}^n\sum_{j\in N_i}[a_{ij}|\xi_i(t) - \xi_j(t)|]\end{aligned}$$

As the graph's communication topology $F(A)$ is connected, we can get that $\sum_{i=1}^n\dot{\Phi}_i(t) = 0$, so $\sum_{i=1}^n\Phi_i(t) = 0$. Thus,

$$\sum_{i=1}^{n}\{[\xi_i(t)-\frac{1}{n}\sum_{k=1}^{n}\xi_k(t)]\times[-\frac{1}{n}\sum_{k=1}^{n}\dot{\xi}_k(t)]\}=0 \tag{10}$$

Therefore, we obtain

$$\dot{V}(t)\leq\frac{\kappa_b(n-1)-\kappa_c}{2}\sum_{i=1}^{n}\sum_{j\in N_i}[a_{ij}\,|\xi_j(t)-\xi_i(t)|]$$

Because $\kappa_c > \kappa_b(n-1)$, and by Lemma 1, we have $\lim_{t\to\infty}|\xi_j(t)-\xi_i(t)| = 0$. This shows that each agent's geometric center estimator based on bearing information can converge in a finite time.According to [3], we can get that $\widetilde{L}_{Ti}(t)$ tends to 0, that is, $\widehat{L}_{Ti}(t)\to L_{Ti}(t)$. According to the above proof, we can get that $\xi_i(t)\to Z^*$.

This part proposes a distributed controller, which is defined as follows:

$$u_i(t)=\lambda\overline{\hat{\psi}}_i(t)+(\Delta-r)\hat{\psi}_i(t) \tag{11}$$

Where $\Delta=\|\xi_i(t)-L_{Ai}(t)\|$, $\hat{\psi}_i=\frac{\xi_i(t)-L_{Ai}(t)}{\Delta}$ and $\overline{\hat{\psi}}_i$ is the perpendicular vector of $\hat{\psi}_i$. λ is the predefined tangential velocity of the agent and r is the radius. Let $\delta_i(t)=\|\varrho_i(t)-\Delta\|$, $\varrho_i(t)=\|Z^*-L_{Ai}(t)\|$. By virtue of the geometric relationship, it is deduced that $\|\varrho_i(t)-\Delta\|\leq\left\|\widetilde{\xi}_i(t)\right\|=\|\xi_i(t)-Z^*\|$. Furthermore, based on the proof provided above, it is established that $\left\|\widetilde{\xi}_i(t)\right\|=\|\xi_i(t)-Z^*\|\to 0$. Subsequently, it is feasible to get that $\varrho_i(t)\to\Delta$. By referring to [3], we can obtain that $\Delta\to r$. Ultimately, we can obtain that $\varrho_i(t)\to r$. Under the controller (11), the agent can finally complete the enclosing formation for multiple targets.

IV. Simulation Results

We simulate and validate our proposed methodologies to show their effectiveness. This simulation setup a MAS consisting of two agents ($n = 2$) and four targets ($m = 4$) on a plane R^2. 2 shows its communication topology. First, we set the initial position of targets as $L_{T1} = [1, 1]^T$, $L_{T2} = [-1, 1]^T$, $L_{T3} = [-1, -1]^T$, $L_{T4} = [1, -1]^T$. Therefore, the geometric center of these targets are $[0, 0]^T$. Besides we set $L_{A1} = [0.2, 2]^T$, $L_{A2} = [0.2, -2]^T$ as the initial positions of agents. The expected circular formation is defined as $r = 2$, $\lambda = 0.3$. The parameters in both estimators and controller are $k_c = 5$, $k_e = 5.5$. Trajectories of agents $L_{A1}(t)$ and $L_{A2}(t)$ are shown in Fig4 and 5. Geometric center estimators $\xi_1(t)$, $\xi_2(t)$ based on bearing information converge to the vicinity of the target center with time, as shown in Fig6 and 7 below. The overall motion trajectory of the agent is shown in Fig3. We can draw from the results that our proposed estimators can finally converge to the real geometrical center of the targets. We can also observe the chattering phenomenon during the estimation. The non-continuous term in the consensus estimator equation (6) is the underlying cause. Nevertheless, we can see from trajectories that the agents can finally achieve a circularly enclosing formation with predefined parameters with only bearing measurement.

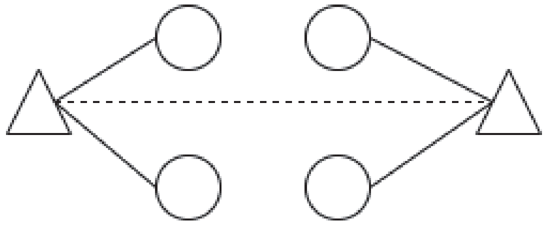

Fig. 2: Observation and Communication topology diagram of the system in the simulation, where the triangles and circles represents the agents and targets respectively. The solid lines are the observations from agent to target and the dashed line are the communication link from between agents.

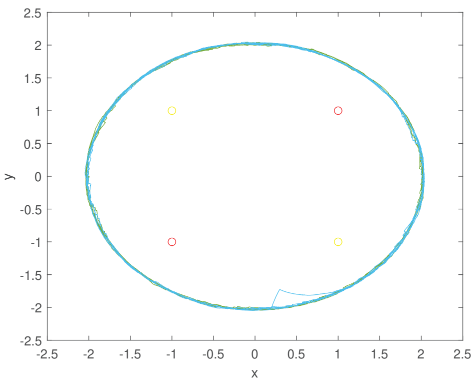

Fig. 3: Motion trajectory of agent, where the blue line denotes the trajectory of agent L_{A1} , the green line represents the trajectory of agent L_{A2} , and the small red and yellow circles represent the target.

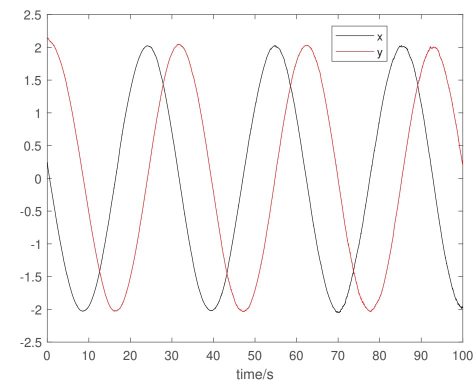

Fig. 4: Trajectory of Agent $L_{A1}(t)$.

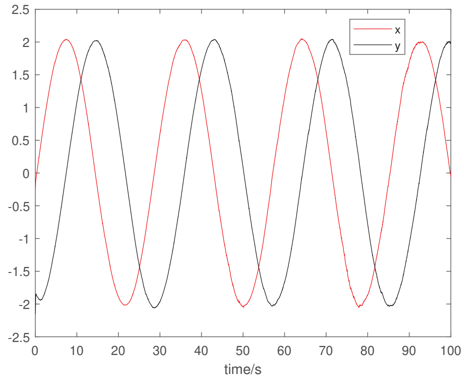

Fig. 5: Trajectory of Agent $L_{A2}(t)$.

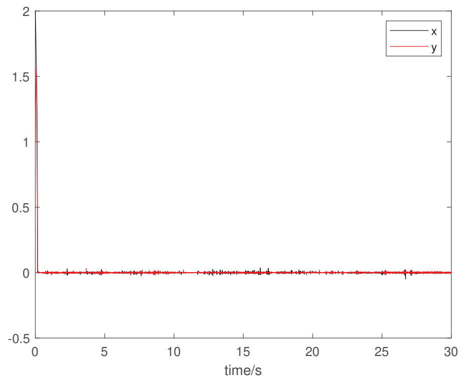

Fig. 6: The estimator $\xi_1(t)$ converges with time.

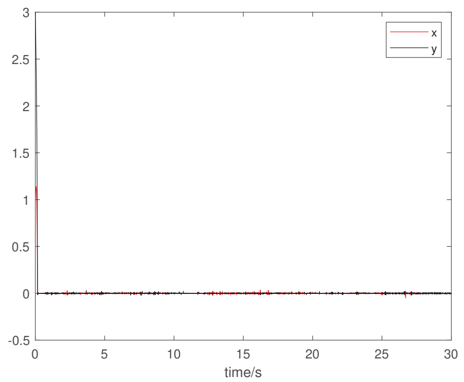

Fig. 7: The estimator $\xi_2(t)$ converges with time.

V. Conclusions

In this paper, our primary research focus pertains to investigating the enclosing control problem for MAS utilizing exclusively bearing measurements for multiple targets. To tackle these issues, we utilize a distributed geometric center estimator that relies on bearing data from multiple targets. On this basis, the agents can be controlled by fixing the origin of circular formation at the geometric center and maintain an expected rotating speed and radius. Our approach's effectiveness is demonstrated through both theoretical analysis and simulation results. Our future works involves extending our results to moving targets, analysing the influence of estimated heading vector $\hat{\psi}_i$ to the control process, and improving the robustness to sensor failure and so on.

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