H_2 Performance Analysis and Distributed H_2 Control of Interconnected Large-Scale Systems

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***Abstract*—This article considers the H_2 performance analysis and distributed H_2 control problems of interconnected large-scale systems (LSSs). According to the developed space construction approach, both novel necessary and sufficient conditions are established to ensure the asymptotic stability and H_2 performance of the studied LSSs. Next, these investigated results are utilized to derive some sufficient conditions for the existence of the H_2 distributed controllers. The desired distributed controller gains are constructed via the solutions of some developed linear matrix inequality conditions, which guarantee the stability and the H_2 performance of the closed-loop LSSs. Finally, three examples are introduced to show the superiority and validity of the proposed theoretical method.**

***Index Terms*—Distributed control, H_2 performance, large-scale systems (LSSs), linear matrix inequalities.**

I. Introduction

Large-scale systems (LSSs) typically comprise several subsystems, which are interconnected over their dynamics. There are numerous engineering examples of LSSs, such as smart power grids [1], spacecraft formations [2], autonomous vehicles systems [3], and traffic systems [4]. When dealing with the control issues of LSSs, the traditional centralized control strategy suffers from heavy communication and computational burdens, and decentralized control sometimes cannot achieve satisfied system performance [5]. As a result, distributed control technology for LSSs has attracted extensive research attention for decades [6], [7], [8], [9], [10], [11], [12], [13].

On the other hand, H_2 optimal control, which is of interest in many control engineering applications, has been heavily studied over the past years [9]. For instance, the authors in [9] considered the interconnected systems which are interconnected over an arbitrary graph, and a modified cone complementary linearization algorithm (MCCL) was proposed to acquire the optimal H_2 performance and the distributed controllers. The authors in [10] introduced an approach to design H_2 robust distributed controllers for interconnected systems with topology uncertainties. In [11], the authors considered the controller design problems for spatially varying interconnected LSSs, which are distributed in one spatial dimension. An efficient iterative algorithm was provided in [11] for stability analysis and the H_2 performance analysis. Recently, the H_2 distributed control problems of "decomposable systems" (which are a particular class of LSSs) were researched in [12]. According to the investigated "decomposition approach" in [12], the authors in [12] designed the distributed controllers which have the same interconnection relation as subsystems to stabilize the considered closed-loop LSSs. Based on the results in [12], distributed H_2 control issues of dynamic multiagent systems were studied in [13].

However, although there are some developed results about distributed H_2 control in the above papers, it is worth noting that the application of the results is limited. For instance, in [9], the subsystems' interconnection matrix is assumed to be a permutation matrix. In addition, the MCCL algorithm proposed in [9] is not suitable for LSSs due to its heavy computational burdens. The authors of [11] designed the optimal H_2 distributed controllers only for spatially varying interconnected systems which are distributed in one spatial dimension. On the other hand, the results of [12] and [13] only can be used to deal with the H_2 distributed control issues for the particular LSSs called "decomposable systems." There are some structure constraints imposing on the system matrices of "decomposable systems," which restrict the application of the results in [12] and[13]. Motivated by the above discussions, how to derive computationally attractive distributed H_2 controller design approaches for more general LSSs is challenging and significant.

This article aims to develop computationally attractive approaches of H_2 performance analysis and distributed H_2 controller design for general LSSs. By using the developed space construction method, we have developed both novel necessary and sufficient linear matrix inequality (LMI) conditions such

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that the studied LSS is asymptotically stable and the desired H_2 performance index is achieved. In order to relieve the burden of computation, other sufficient lower dimensional LMI conditions are also provided. Then, the developed LMI conditions are utilized to design the distributed controllers. The solutions of the LMIs can be used to construct the distributed H_2 controller gains. In the last, three examples are introduced to illustrate the validity and superiority of the proposed theoretical method. One example shows that compared with the method given by [12], our method achieves better system performance in the situation of various system matrix parameters. The second example illustrates that our results are applied to design the distributed H_2 controllers for more general LSSs. The considered LSS in the second example consists of 500 subsystems. The last example compares the computation time cost by our method and the methods in [12], [14], and [15]. This example shows that the computation time cost by our developed method is always less than those methods in [12], [14], and [15], which means that our developed controller design method is more computationally attractive. We summarize the main contributions of this article as follows.

1) Novel (both necessary and sufficient) conditions are established such that the studied LSS is asymptotically stable with a given H_2 performance index. If one uses the classical necessary and sufficient conditions given by [14] and [15] to check the system stability and H_2 performance, the computational cost is heavy because there are several matrix inversion terms that need to be solved. However, our novel conditions are more computationally attractive due to the elimination of the large dimensional matrix inversion terms.
2) The developed theoretical results can be utilized to design distributed controllers for more general LSSs. The authors of [9] only considered the situation that the subsystems' interconnection matrix is a permutation matrix which need to satisfy a certain constraint. The authors of [11] designed the distributed H_2 controllers only for spatially varying LSSs where the subsystems were distributed in one spatial dimension. The authors of [12] and [13] studied distributed H_2 control issues for the particular "decomposable systems," where the system matrices in [12] and [13] need to satisfy the predefined property. However, this article does not require the above constraints and assumptions.

Notations: $\mathbb{N}^+, \mathbb{R}^{n\times m}, \mathbb{R}^n_s$ represent the set of positive integers, the matrices of dimension $n \times m$, and the symmetric matrices of dimension $n \times n$. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $X < 0$ ($X \leq 0$) means X is negative definite (negative semidefinite). $\mathrm{in}_-\{A\}$ denotes the number of negative eigenvalues of X. The notation $\mathrm{col}\{x^i|_{i=1}^N\}$ means the vector $[x^1, x^2, \ldots, x^N]^\top$. The symbol $\mathrm{diag}\{X_i|_{i=1}^N\}$ means the diagonal block matrix with diagonal elements $X_1, X_2, \ldots, X_N$. The kernel and the image of X are represented by $\ker\{X\}$ and $\mathrm{im}\{X\}$, respectively. The symbols X^{oc} and X_{bk} mean the orthogonal complement of X and the basis for $\ker\{X\}$, respectively. $\dim\{\mathrm{im}\{X\}\}$ is used to denote the dimension of $\mathrm{im}\{X\}$. The summation notation like $\sum_{i=1}^N n_{vi}$ is abbreviated as S_v. On the other hand, $Y^\top XY$ and $Z^\top Y^\top XYZ$ are expressed as $(\star)^\top XY$ and $(\star\star)^\top XYZ$. For subspace $\mathscr{B} \subset \mathbb{R}^n$, any S with full column rank and $\mathscr{B} = \mathrm{im}\{S\}$ is said to be a basis matrix of $\mathscr{B}$. $A > 0$ ($A < 0$) on $\mathscr{B}$ means that $S^\top AS > 0$ ($S^\top AS < 0$).

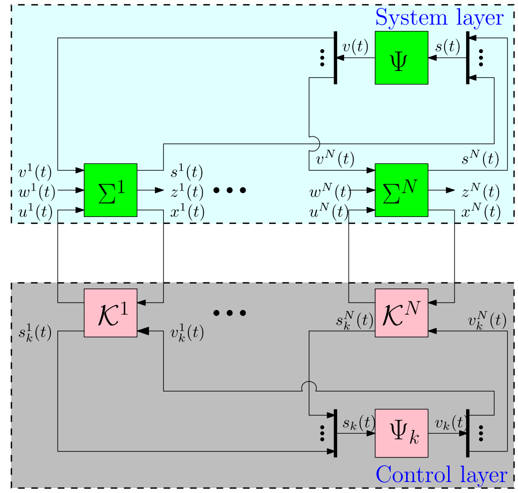

Fig. 1. System structure of interconnected LSSs.

II. Problem Statement

Consider a LSS with N interconnected subsystems (see Fig. 1). Each subsystem Σ^i is controlled by the local subcontroller $\mathcal{K}^i$ ($i \in \Sigma \triangleq \{1, 2, \ldots, N\}$). The ith ($i \in \Sigma$) subsystem is described as

$$\begin{bmatrix} x^i(t+1) \\ s^i(t) \\ z^i(t) \end{bmatrix} = \begin{bmatrix} A^i_{xx} & A^i_{xv} & B^i_{xw} & B^i_{xu} \\ A^i_{sx} & A^i_{sv} & 0 & B^i_{su} \\ C^i_{zx} & C^i_{zv} & 0 & D^i_{zu} \end{bmatrix} \begin{bmatrix} x^i(t) \\ v^i(t) \\ w^i(t) \\ u^i(t) \end{bmatrix} \tag{1}$$

where $x^i(t) \in \mathbb{R}^{n_{xi}}, z^i(t) \in \mathbb{R}^{n_{zi}}, w^i(t) \in \mathbb{R}^{n_{wi}}$, and $u^i(t) \in \mathbb{R}^{n_{ui}}$ ($i \in \Sigma$) are used to represent the state, the performance output, the external disturbance, and the control of the ith ($i \in \Sigma$) subsystem. For the ith subsystem in the system layer, $s^i(t) \in \mathbb{R}^{n_{si}}$ means the output signals toward other subsystems and $v^i(t) \in \mathbb{R}^{n_{vi}}$ means the input from other subsystems. In this article, all subsystems' interconnection relation is given by $\Psi \in \mathbb{R}^{S_v \times S_s}$ as

$$v(t) = \Psi s(t) \tag{2}$$

where $v(t) = \mathrm{col}\{v^i(t)|_{i=1}^N\}$ and $s(t) = \mathrm{col}\{s^i(t)|_{i=1}^N\}$.

In the control layer, the notations $s^i_k \in \mathbb{R}^{n_{si}}$ and $v^i_k \in \mathbb{R}^{n_{vi}}$ represent the subcontrollers' interconnection signals. The interconnection relation among subcontrollers is given by Ψ_k. The subscript "k" in the signals of the control layer is used to distinguish the signals in the control layer with the signals in the system layer.

Remark 1: Compared with the existing literature [9], [11], [12], [13], the studied LSS (1)–(2) are more general. For example, the interconnection matrix Ψ was assumed to be a permutation matrix in [9]. The authors of [11] studied the H_2 distributed control issues for interconnected LSSs where the subsystems are distributed in one spatial dimension. The authors of [12] and [13] only studied the particular "decomposable systems." These assumptions and constraints are not required in this article.

Denote that

$$x(t)=\mathrm{col}\{x^i(t)|_{i=1}^N\},\ z(t)=\mathrm{col}\{z^i(t)|_{i=1}^N\},$$
$$u(t)=\mathrm{col}\{u^i(t)|_{i=1}^N\},\ w(t)=\mathrm{col}\{w^i(t)|_{i=1}^N\},$$
$$A_{*\#}=\mathrm{diag}\{A^i_{*\#}|_{i=1}^N\},\ B_{*\#}=\mathrm{diag}\{B^i_{*\#}|_{i=1}^N\},$$
$$C_{*\#}=\mathrm{diag}\{C^i_{*\#}|_{i=1}^N\},\ D_{*\#}=\mathrm{diag}\{D^i_{*\#}|_{i=1}^N\}$$

where $*, \# \in \{x, v, w, u, s, z\}$.

With the above notations, the state space representation (1) is rewritten in the following compact form:

$$\begin{bmatrix} x(t+1) \\ s(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xv} & B_{xw} & B_{xu} \\ A_{sx} & A_{sv} & 0 & B_{su} \\ C_{zx} & C_{zv} & 0 & D_{zu} \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \\ w(t) \\ u(t) \end{bmatrix}. \quad (3)$$

In this article, we consider that the LSS is well posed [14], [16], [17] and assume that $I - \Psi A_{sv}$ is invertible. Thus, the term $I - A_{sv}\Psi$ is also invertible. We can also see the similar assumptions in [16] and [17].

From the fact that $(I - \Psi A_{sv})\Psi = \Psi(I - A_{sv}\Psi)$, we have

$$\Psi(I - A_{sv}\Psi)^{-1} = (I - \Psi A_{sv})^{-1}\Psi. \quad (4)$$

From (2)–(4), the considered LSS (3) can be expressed as

$$\begin{cases} x(t+1) = A(\Psi)x(t) + E(\Psi)u(t) + B_{xw}w(t), \\ z(t) = C(\Psi)x(t) + F(\Psi)u(t) \end{cases} \quad (5)$$

where

$$A(\Psi) = A_{xx} + A_{xv}(I - \Psi A_{sv})^{-1}\Psi A_{sx}$$
$$E(\Psi) = B_{xu} + A_{xv}(I - \Psi A_{sv})^{-1}\Psi B_{su}$$
$$C(\Psi) = C_{zx} + C_{zv}(I - \Psi A_{sv})^{-1}\Psi A_{sx}$$
$$F(\Psi) = D_{zu} + C_{zv}(I - \Psi A_{sv})^{-1}\Psi B_{su}.$$

Let the symbol $T_{wz}(\zeta)$ denote the transfer function from $w(t)$ to $z(t)$ and $\|T_{wz}(\zeta)\|_2$ represents the H_2-norm of $T_{wz}(\zeta)$. The objectives of this article are summarized as follows.

1) For the open-loop LSS (5) ($u(t) \equiv 0$), develop both novel necessary and sufficient conditions for ensuring its exponential stability and the desired H_2 performance, i.e., $\|T_{wz}(\zeta)\|_2 < \gamma$, where $\gamma > 0$ is a given positive scalar.
2) Design distributed controllers for LSS (5) such that the closed-loop system is stable with the prescribed H_2 performance.

III. Stability Analysis and H_2 Performance Analysis

In this section, both necessary and sufficient conditions are investigated for LSS (5) with $u(t) \equiv 0$ such that it is stable and the desired H_2 performance index is achieved. Furthermore, in order to reduce the computational burden, some computationally attractive sufficient LMI conditions are investigated to check the stability and the H_2 performance of system (5).

Lemma 1: ([14]) Given $\gamma > 0$, LSS (5) with $u(t) \equiv 0$ is stable and satisfies the desired H_2 performance $\|T_{wz}(\zeta)\|_2 < \gamma$, if and only if there exists $X > 0$ ($X \in \mathbb{R}_s^{S_x}$) such that

$$A(\Psi)^\top XA(\Psi) - X + C(\Psi)^\top C(\Psi) < 0 \quad (6)$$
$$\mathrm{Trace}(B_{xw}^\top XB_{xw}) < \gamma^2. \quad (7)$$

Lemma 2: Denote the subspace of $\mathbb{R}^n$ as $\mathscr{B}$, $V \in \mathbb{R}^{q\times n}$, and $Q \in \mathbb{R}_s^n$. Suppose that the subspace $\mathscr{S}(\Delta) \subset \mathbb{R}^q$ depends on the matrix parameter Δ continuously. Define that $\mathscr{B}(\Delta) \triangleq \{\zeta | \zeta \in \mathscr{B}, V\zeta \in \mathscr{S}(\Delta)\}$. Then, the inequality

$$Q < 0 \text{ on } \mathscr{B}(\Delta)$$

is satisfied, if and only if the following two inequalities hold:

$$\begin{cases} Q + V^\top PV < 0 \text{ on } \mathscr{B}, \\ P > 0 \text{ on } \mathscr{S}(\Delta) \end{cases}$$

where $P \in \mathbb{R}_s^q$.

Proof: One can find the strict and rigorous proof in [18, Th. 8]. In [18, Th. 8], there is a matrix parameter set $\boldsymbol{\Delta}$ and a matrix variable U. Here, we consider that there is only one element "Δ" in the set "$\boldsymbol{\Delta}$" and let the matrix variable "$U \triangleq I$," [18, Th. 8] be reformulated in Lemma 2. ■

Lemma 1 provides necessary and sufficient conditions to evaluate the H_2 performance of LSS (5). However, the conditions in Lemma 1 are computationally expensive because of the inversion term $(I - \Psi A_{sv})^{-1}$ in $A(\Psi)$ and $C(\Psi)$. Besides, the inversion term may even bring numeric unstable problems in the situation of high dimensions [19], [20], [21]. Therefore, Lemma 1 is not appropriate to be utilized for LSSs. To deal with this difficulty as follows, novel necessary and sufficient conditions are derived to check the stability and the H_2 performance of LSS (5) with $u(t) \equiv 0$. The novel conditions avoid the inversion computation of large-dimensional matrices.

Theorem 1: Given a scalar $\gamma > 0$ and $u(t) \equiv 0$, the considered LSS (5) is stable and has the H_2 performance $\|T_{wz}(\zeta)\|_2 < \gamma$, if and only if there exist $X > 0$ ($X \in \mathbb{R}_s^{S_x}$), $Q \in \mathbb{R}_s^{S_s}$, $S \in \mathbb{R}^{S_s\times S_v}$ and $R \in \mathbb{R}_s^{S_v}$ that satisfy

$$(\star)^\top \begin{bmatrix} X & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -X & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & M & S & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & S^\top & R & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix} \begin{bmatrix} A_{xx} & A_{xv} \\ I & \mathbf{0} \\ A_{sx} & A_{sv} \\ \mathbf{0} & I \\ C_{zx} & C_{zv} \end{bmatrix} < 0, \quad (8)$$

$$(\star)^\top \begin{bmatrix} M & S \\ S^\top & R \end{bmatrix} \begin{bmatrix} I \\ \Psi \end{bmatrix} > 0, \quad (9)$$

$$\mathrm{Trace}(B_{xw}^\top XB_{xw}) < \gamma^2. \quad (10)$$

Proof: First, we introduce the space constructions as follows:

$$\mathscr{B} = \mathrm{im}\{\bar{\mathscr{B}}\}, \quad \mathscr{S}(\Psi) = \ker\{[-\Psi \quad I]\} \quad (11)$$

where $\bar{\mathscr{B}} = \begin{bmatrix} A_{xx} & A_{xv} \\ I & \mathbf{0} \\ A_{sx} & A_{sv} \\ \mathbf{0} & I \\ C_{zx} & C_{zv} \end{bmatrix}$.

Then, we have

$$\mathscr{S}(\Psi) = \ker\{[-\Psi \quad I]\} = \mathrm{im}\left\{\begin{bmatrix} I \\ \Psi \end{bmatrix}\right\}. \tag{12}$$

Define that

$$Q = \begin{bmatrix} X & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -X & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix}, \quad \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \tag{13}$$

$$V = \begin{bmatrix} \mathbf{0} & \mathbf{0} & I & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I & \mathbf{0} \end{bmatrix}, \quad P = \begin{bmatrix} M & S \\ S^\top & R \end{bmatrix}. \tag{14}$$

With the above notations, the term $\mathscr{B}(\Psi) = \{\zeta | \zeta \in \mathscr{B}, V\zeta \in \mathscr{S}(\Psi)\}$ is written in the form

$$\begin{aligned} \mathscr{B}(\Psi) &= \{\zeta | \zeta = \bar{\mathscr{B}}\xi, [-\Psi \quad I]V\bar{\mathscr{B}}\xi = 0\} \\ &= \{\zeta | \zeta = \bar{\mathscr{B}}\xi, (I - \Psi A_{sv})\xi_2 - \Psi A_{sx}\xi_1 = 0\} \\ &= \left\{\zeta | \zeta = \bar{\mathscr{B}}\begin{bmatrix} \xi_1 \\ (I - \Psi A_{sv})^{-1}\Psi A_{sx}\xi_1 \end{bmatrix}\right\} \\ &= \left\{\zeta | \zeta = \begin{bmatrix} A(\Psi) \\ I \\ A_{sx} + A_{sv}(I - \Psi A_{sv})^{-1}\Psi A_{sx} \\ (I - \Psi A_{sv})^{-1}\Psi A_{sx} \\ C(\Psi) \end{bmatrix} \xi_1 \right\} \\ &= \mathrm{im}\left\{\begin{bmatrix} A(\Psi) \\ I \\ A_{sx} + A_{sv}(I - \Psi A_{sv})^{-1}\Psi A_{sx} \\ (I - \Psi A_{sv})^{-1}\Psi A_{sx} \\ C(\Psi) \end{bmatrix}\right\}. \end{aligned}$$

From (11)–(14), it can be verified that (8)–(9) are equivalent to

$$Q + V^\top PV < 0 \,\mathrm{on}\, \mathscr{B}, \tag{15}$$

$$P > 0 \,\mathrm{on}\, \mathscr{S}(\Psi). \tag{16}$$

According to Lemma 2, (15)–(16) are equivalent to

$$Q < 0 \,\mathrm{on}\, \mathscr{B}(\Psi). \tag{17}$$

By direct computations, (17) implies that

$$A(\Psi)^\top XA(\Psi) - X + C(\Psi)^\top C(\Psi) < 0. \tag{18}$$

Here, the inequality (18) is exactly (6) in Lemma 1.

We have proven that (8)–(9) $\Leftrightarrow$ (15)–(16) $\Leftrightarrow$ (17) $\Leftrightarrow$ (18) $\Leftrightarrow$ (6). Note that (10) is exactly (7) in Lemma 1. Therefore, LSS (5) with $u(t) \equiv 0$ is stable with $\|T_{wz}(\zeta)\|_2 < \gamma$, if and only if (8)–(10) are satisfied. ■

Remark 2: In Theorem 1, (8)–(10) are necessary and sufficient conditions for ensuring the stability and the desired H_2 performance index of the open-loop LSS (5) ($u(0) \equiv 0$). Compared to conditions (6)–(7), one can use (8)–(10) to check the stability and the H_2 performance of system (5) without the matrix inversion computation.

Note that in the left-hand side of (8), the dimensions increase with the subsystems number N because of the block diagonal structure of $A_{*\#}$ and $C_{*\#}$ ($*, \# \in \{x, v, s, z\}$). In order to avoid solving LMI conditions with the dimension scaled by subsystems number N, sufficient LMI conditions with lower dimensions are further derived to guarantee the stability and the H_2 performance of LSS (5) with $u(t) \equiv 0$.

Theorem 2: Given $\gamma > 0$, if there exist $X^i > 0$ ($X^i \in \mathbb{R}^{n_{xi}}_s$), $Q^i \in \mathbb{R}^{n_{si}}_s$, $S^i \in \mathbb{R}^{n_{si} \times n_{vi}}$, and $R^i \in \mathbb{R}^{n_{vi}}_s$ ($i \in \Sigma$) such that

$$(\star)^\top \begin{bmatrix} X^i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -X^i & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & M^i & S^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (S^i)^\top & R^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix} \begin{bmatrix} A^i_{xx} & A^i_{xv} \\ I & \mathbf{0} \\ A^i_{sx} & A^i_{sv} \\ \mathbf{0} & I \\ C^i_{zx} & C^i_{zv} \end{bmatrix} < 0 \tag{19}$$

$$(\star)^\top \begin{bmatrix} \mathrm{diag}\{M^i|^N_{i=1}\} & \mathrm{diag}\{S^i|^N_{i=1}\} \\ \mathrm{diag}\{(S^i)^\top|^N_{i=1}\} & \mathrm{diag}\{R^i|^N_{i=1}\} \end{bmatrix} \begin{bmatrix} I \\ \Psi \end{bmatrix} > 0 \tag{20}$$

$$\sum_{i=1}^N \mathrm{Trace}((B^i_{xw})^\top X^i B^i_{xw}) < \gamma^2 \tag{21}$$

are feasible for every $i \in \Sigma$, then the open-loop LSS (5) ($u(t) \equiv 0$) is stable and has the H_2 performance $\|T_{wz}(\zeta)\|_2 < \gamma$.

Proof: According to (19), we define that

$$\begin{bmatrix} \Delta^i_{11} & \Delta^i_{12} \\ (\Delta^i_{12})^\top & \Delta^i_{22} \end{bmatrix} \triangleq (\star)^\top \begin{bmatrix} X^i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -X^i & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & M^i & S^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (S^i)^\top & R^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix} \times \begin{bmatrix} A^i_{xx} & A^i_{xv} \\ I & \mathbf{0} \\ A^i_{sx} & A^i_{sv} \\ \mathbf{0} & I \\ C^i_{zx} & C^i_{zv} \end{bmatrix} < 0$$

where

$$\begin{aligned} \Delta^i_{11} &= (A^i_{xx})^\top X^i A^i_{xx} - X^i + (A^i_{sx})^\top M^i A^i_{sx} + (C^i_{zx})^\top C^i_{zx}, \\ \Delta^i_{12} &= (A^i_{xx})^\top X^i A^i_{xv} + (A^i_{sx})^\top M^i A^i_{sv} + (A^i_{sx})^\top S^i + (C^i_{zx})^\top C^i_{zv}, \\ \Delta^i_{22} &= (A^i_{xv})^\top X^i A^i_{xv} + (A^i_{sv})^\top M^i A^i_{sv} + (S^i)^\top A^i_{sv} + (A^i_{sv})^\top S^i \\ &\quad + R^i + (C^i_{zv})^\top C^i_{zv}. \end{aligned}$$

Therefore

$$\mathrm{diag}\left\{\begin{bmatrix} \Delta^i_{11} & \Delta^i_{12} \\ (\Delta^i_{12})^\top & \Delta^i_{22} \end{bmatrix}|^N_{i=1}\right\} < 0. \tag{22}$$

Let

$$X = \mathrm{diag}\{X^i|_{i=1}^N\}, \tag{23}$$

$$\begin{bmatrix} M & S \\ S^\top & R \end{bmatrix} = \begin{bmatrix} \mathrm{diag}\{M^i|_{i=1}^N\} & \mathrm{diag}\{S^i|_{i=1}^N\} \\ \mathrm{diag}\{(S^i)^\top|_{i=1}^N\} & \mathrm{diag}\{R^i|_{i=1}^N\} \end{bmatrix}. \tag{24}$$

Noting the block diagonal matrices $A_{*\#} = \mathrm{diag}\{A^i_{*\#}|_{i=1}^N\}$ and $C_{*\#} = \mathrm{diag}\{C^i_{*\#}|_{i=1}^N\}$ $(*, \# \in \{x, v, s, z\})$, one, therefore, obtains for (8) that

$$(\star)^\top \begin{bmatrix} X & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -X & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & M & S & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & S^\top & R & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix} \begin{bmatrix} A_{xx} & A_{xv} \\ I & \mathbf{0} \\ A_{sx} & A_{sv} \\ \mathbf{0} & I \\ C_{zx} & C_{zv} \end{bmatrix}$$
$$= \begin{bmatrix} \mathrm{diag}\{\Delta^i_{11}|_{i=1}^N\} & \mathrm{diag}\{\Delta^i_{12}|_{i=1}^N\} \\ \mathrm{diag}\{(\Delta^i_{12})^\top|_{i=1}^N\} & \mathrm{diag}\{\Delta^i_{22}|_{i=1}^N\} \end{bmatrix}$$
$$= \Pi_1^\top \mathrm{diag}\left\{ \begin{bmatrix} \Delta^i_{11} & \Delta^i_{12} \\ (\Delta^i_{12})^\top & \Delta^i_{22} \end{bmatrix} |_{i=1}^N \right\} \Pi_1 < 0 \tag{25}$$

where Π_1 is given as

$$\Pi_1 = \left[\begin{array}{cccccccc} I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & I & \mathbf{0} \\ \hline \mathbf{0} & I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & I \end{array}\right]^\top.$$

Here, the first and the second "=" in (25) hold according to direct mathematical manipulations, and the "<" in (25) holds according to (22).

On the other hand, the inequality (9) is satisfied by combining (20) with (24).

Noting the block diagonal matrices B_{xw} and X, we obtain for (10) that

$$\mathrm{Trace}(B_{xw}^\top X B_{xw}) = \sum_{i=1}^N (B^i_{xw})^\top X^i B^i_{xw} < \gamma^2.$$

Here, the "<" holds due to (21).

We have proven that (19)–(21) $\Rightarrow$ (8)–(10) with the matrices X, Q, S, R given by (23)–(24). According to Theorem 1, we give the conclusion that system (5) $(u(t) \equiv 0)$ is stable and has the H_2 performance $\|T_{wz}(\zeta)\|_2 < \gamma$ if (19)–(21) hold. ∎

Remark 3: Compared with Lemma 1, a common feature of Theorems 1 and 2 is the elimination of the inversion terms. The necessary and sufficient conditions in Theorem 1 ensure that the studied system is stable and satisfies the desired H_2 performance index. However, the conditions of Theorem 2 are only sufficient. The advantages of Theorem 2 are that the developed LMI conditions are of lower dimensions and have fewer decision variables, which make the inequalities of Theorem 2 easy to solve. The comparison of computational complexity of different approaches is given in Table I.

IV. H_2 Distributed Controller Design

A. Controller Existence

The local subcontroller $\mathcal{K}^i$ is designed as the form

$$\begin{bmatrix} x^i_k(t+1) \\ s^i_k(t) \\ u^i(t) \end{bmatrix} = \begin{bmatrix} (A^i_{xx})_k & (A^i_{xv})_k & (B^i_x)_k \\ (A^i_{sx})_k & (A^i_{sv})_k & (B^i_s)_k \\ (C^i_x)_k & (C^i_s)_k & D^i_k \end{bmatrix} \begin{bmatrix} x^i_k(t) \\ v^i_k(t) \\ x^i(t) \end{bmatrix} \tag{26}$$

where $x^i_k(t) \in \mathbb{R}^{n_{xi}}$ denotes the ith $(i \in \Sigma)$ subcontroller state, and the symbols $s^i_k \in \mathbb{R}^{n_{si}}$ and $v^i_k \in \mathbb{R}^{n_{vi}}$ $(i \in \Sigma)$ represent the subcontrollers' interconnection signals; see Fig. 1. There is a subscript "k" in the subcontrollers' interconnection signals (s^i_k, v^i_k), which is used to distinguish the subcontrollers' interconnection signals with the subsystems' interconnection signals (s^i, v^i). Here, we consider that all subcontrollers are interconnected through the matrix $\Psi_k \in \mathbb{R}^{S_v \times S_s}$ as

$$v_k(t) = \Psi_k s_k(t) \tag{27}$$

where $v_k(t) = \mathrm{col}\{v^i_k(t)|_{i=1}^N\}$, $s_k(t) = \mathrm{col}\{s^i_k(t)|_{i=1}^N\}$. The matrix Ψ_k stands for the subcontrollers' interconnection relation.

Remark 4: The matrices Ψ_k and Ψ are used to represent the subcontrollers' interconnection relation and the subsystems' interconnection relation, respectively. In this article, Ψ_k and Ψ can be different matrices. Design distributed controllers, where the subcontrollers' interconnection relation are the same as the subsystems' interconnection relation (i.e., $\Psi_k = \Psi$), is a particular case of this article [12], [22].

Combining (1)–(2) with (26)–(27) gives the closed-loop system

$$\left[\begin{array}{c} x^i(t+1) \\ x^i_k(t+1) \\ \hline s^i(t) \\ s^i_k(t) \\ \hline z^i(t) \end{array}\right] = \begin{bmatrix} (A^i_{xx})_c & (A^i_{xv})_c & (B^i_{xw})_c \\ (A^i_{sx})_c & (A^i_{sv})_c & (B^i_{sw})_c \\ (C^i_{zx})_c & (C^i_{zv})_c & (D^i_{zw})_c \end{bmatrix} \left[\begin{array}{c} x^i(t) \\ x^i_k(t) \\ \hline v^i(t) \\ v^i_k(t) \\ \hline w^i(t) \end{array}\right], \tag{28}$$

$$\begin{bmatrix} v(t) \\ v_k(t) \end{bmatrix} = \begin{bmatrix} \Psi & \mathbf{0} \\ \mathbf{0} & \Psi_k \end{bmatrix} \begin{bmatrix} s(t) \\ s_k(t) \end{bmatrix} \tag{29}$$

where

$$\begin{bmatrix} (A^i_{xx})_c & (A^i_{xv})_c \\ (A^i_{sx})_c & (A^i_{sv})_c \\ (C^i_{zx})_c & (C^i_{zv})_c \end{bmatrix} = \mathcal{A}^i + (\mathcal{B}^i)^\top \mathcal{K}^i \mathcal{C}^i \tag{30}$$

$$(B^i_{xw})_c = \begin{bmatrix} B^i_{xw} \\ \mathbf{0} \end{bmatrix}, \quad (B^i_{sw})_c = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (D^i_{zw})_c = \mathbf{0} \tag{31}$$

$$\mathcal{A}^i = \left[\begin{array}{cc|cc} A^i_{xx} & \mathbf{0} & A^i_{xv} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline A^i_{sx} & \mathbf{0} & A^i_{sv} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline C^i_{zx} & \mathbf{0} & C^i_{zv} & \mathbf{0} \end{array}\right], \quad (\mathcal{B}^i)^\top = \left[\begin{array}{ccc} \mathbf{0} & \mathbf{0} & B^i_{xu} \\ I & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & B^i_{su} \\ \mathbf{0} & I & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & D^i_{zu} \end{array}\right]$$

TABLE I
COMPARISON OF COMPUTATIONAL COMPLEXITY FOR DIFFERENT METHODS

	Size of LMIs	Number of decision variables
Lemma 1 (with matrix inversion computation)	$\Sigma_{i=1}^{N} n_{xi}$	$\frac{\Sigma_{i=1}^{N} n_{xi}(\Sigma_{i=1}^{N} n_{xi}+1)}{2}$
Theorem 1 (without matrix inversion computation)	$\Sigma_{i=1}^{N}(n_{xi}+n_{vi})$	$\frac{\Sigma_{i=1}^{N} n_{xi}(\Sigma_{i=1}^{N} n_{xi}+1)}{2}+\frac{\Sigma_{i=1}^{N} n_{si}(\Sigma_{i=1}^{N} n_{si}+1)}{2}+\frac{\Sigma_{i=1}^{N} n_{vi}(\Sigma_{i=1}^{N} n_{vi}+1)}{2}+\Sigma_{i=1}^{N} n_{vi}\Sigma_{i=1}^{N} n_{si}$
Theorem 2 (without matrix inversion computation)	$n_{xi}+n_{vi}$	$\frac{n_{xi}(n_{xi}+1)}{2}+\frac{n_{si}(n_{si}+1)}{2}+\frac{n_{vi}(n_{vi}+1)}{2}+n_{vi}n_{si}$

$$\mathcal{K}^i=\begin{bmatrix}(A^i_{xx})_k & (A^i_{xv})_k & (B^i_{x})_k\\ (A^i_{sx})_k & (A^i_{sv})_k & (B^i_{s})_k\\ (C^i_{x})_k & (C^i_{s})_k & D^i_k\end{bmatrix},\ \mathcal{C}^i=\left[\begin{array}{cc|cc}\mathbf{0} & I & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I\\ I & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{array}\right]. \tag{32}$$

Remark 5: It is worth noting that in (30), the matrix $\mathcal{K}^i$ $(i\in\Sigma)$ consists of all unknown controller parameters, and $\mathcal{A}^i,\mathcal{B}^i$, and $\mathcal{C}^i$ $(i\in\Sigma)$ are known system matrices. The form of the right-hand side of (30) makes it convenient to design the distributed controllers as follows.

Theorem 3: Given $\gamma>0$, the closed-loop system (28)–(29) is stable and the prescribed H_2 performance is satisfied, if there exist controller parameters in $\mathcal{K}^i$ $(i\in\Sigma)$, $\mathcal{X}^i>0,(\mathcal{X}^i\in\mathbb{R}^{2n_{xi}}_s)$ $\mathcal{M}^i\in\mathbb{R}^{2n_{si}}_s,\mathcal{S}^i\in\mathbb{R}^{2n_{si}\times 2n_{vi}}$, and $\mathcal{R}^i\in\mathbb{R}^{2n_{vi}}_s$ $(i\in\Sigma)$ such that (33)–(35) are feasible for every $i\in\Sigma$

$$(\star)^\top T^iN^i<0, \tag{33}$$

$$(\star)^\top\begin{bmatrix}\operatorname{diag}\{\mathcal{M}^i|_{i=1}^N\} & \operatorname{diag}\{\mathcal{S}^i|_{i=1}^N\}\\ \operatorname{diag}\{(\mathcal{S}^i)^\top|_{i=1}^N\} & \operatorname{diag}\{\mathcal{R}^i|_{i=1}^N\}\end{bmatrix}\begin{bmatrix}I & \mathbf{0}\\ \mathbf{0} & I\\ \Psi & \mathbf{0}\\ \mathbf{0} & \Psi_k\end{bmatrix}>0 \tag{34}$$

$$\sum_{i=1}^N \operatorname{Trace}((B^i_{xw})_c^\top\mathcal{X}^i(B^i_{xw})_c)<\gamma^2 \tag{35}$$

where

$$T^i=\begin{bmatrix}\mathcal{X}^i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mathcal{X}^i & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathcal{M}^i & \mathcal{S}^i & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & (\mathcal{S}^i)^\top & \mathcal{R}^i & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I\end{bmatrix}$$
$$N^i=\begin{bmatrix}(A^i_{xx})_c & (A^i_{xv})_c\\ I & \mathbf{0}\\ (A^i_{sx})_c & (A^i_{sv})_c\\ \mathbf{0} & I\\ (C^i_{zx})_c & (C^i_{zv})_c\end{bmatrix}. \tag{36}$$

Proof: The results are easily obtained by applying Theorem 2 to the closed-loop system (28)–(29). ■

Note that $\mathcal{X}^i,\mathcal{M}^i,\mathcal{S}^i,\mathcal{R}^i$ $(i\in\Sigma)$ and the controller parameters in $\mathcal{K}^i$ are unknown matrix variables. The inequality (33) is nonlinear. As follows, first, LMI conditions without controller parameters are established to guarantee the stability and the desire H_2 index of the system (28)–(29). Then, the solutions of the investigated LMI conditions are utilized to construct the controller gains in $\mathcal{K}^i$ $(i\in\Sigma)$.

Lemma 3: If there exist compatible dimensional matrices $F<0,H>0$, and E, then we have the following two properties:

1) $\left[\begin{array}{c|c}H & E\\ \hline E^\top & F\end{array}\right]^{-1}=\left[\begin{array}{c|c}\Upsilon^{-1} & -\Upsilon^{-1}EF^{-1}\\ \hline -F^{-1}E^\top\Upsilon^{-1} & F^{-1}+F^{-1}E^\top\Upsilon^{-1}EF^{-1}\end{array}\right]$, where $\Upsilon=(H-EF^{-1}E^\top)>0$;
2) $\text{in}_-\left\{\begin{bmatrix}H & E\\ E^\top & F\end{bmatrix}\right\}=\text{in}_-\{F\}$.

Proof: One can see the proof of the property 1) in [14, Sec. 2.3]. We now consider the second property. It is easy to verify that

$$\begin{bmatrix}H & E\\ E^\top & F\end{bmatrix}=\begin{bmatrix}I & \mathbf{0}\\ F^{-1}E^\top & I\end{bmatrix}^\top\begin{bmatrix}\Upsilon & \mathbf{0}\\ \mathbf{0} & F\end{bmatrix}\begin{bmatrix}I & \mathbf{0}\\ F^{-1}E^\top & I\end{bmatrix}$$

which implies that

$$\text{in}_-\left\{\begin{bmatrix}H & E\\ E^\top & F\end{bmatrix}\right\}=\text{in}_-\left\{\begin{bmatrix}\Upsilon & \mathbf{0}\\ \mathbf{0} & F\end{bmatrix}\right\}=\text{in}_-\{F\}.$$

Here, the second "=" is satisfied due to $\Upsilon>0$ and $F<0$. ■

Lemma 4: ([18], [23]) Given an invertible matrix Z, if the number of negative eigenvalues of Z is equal to the dimension of the image of Y, i.e., $\text{in}_-\{Z\}=\dim\{\text{im}\{Y\}\}$, then

$$Y^\top ZY<0\Leftrightarrow(Y^{oc})^\top Z^{-1}Y^{oc}>0$$

where Y^{oc} is used to represent the orthogonal complement of Y.

Lemma 5: ([23]) For the unknown matrix Ω, the following inequality

$$\begin{bmatrix}I\\ \Gamma+\Lambda^\top\Omega\Phi\end{bmatrix}^\top N\begin{bmatrix}I\\ \Gamma+\Lambda^\top\Omega\Phi\end{bmatrix}<0$$

is feasible if and only if

$$(\star)^\top N\begin{bmatrix}\Phi_{bk}\\ \Gamma\Phi_{bk}\end{bmatrix}<0,\quad(\star)^\top N^{-1}\begin{bmatrix}-\Gamma^\top\Lambda_{bk}\\ \Lambda_{bk}\end{bmatrix}>0$$

hold. Here, Φ_{bk} and Λ_{bk} are used to denote a basis for the kernel of Φ and Λ. The matrices N and N^{-1} are given by

$$N=\begin{bmatrix}N^{11} & N^{12}\\ (N^{12})^\top & N^{22}\end{bmatrix},\quad N^{-1}=\begin{bmatrix}N^{11}_d & N^{12}_d\\ (N^{12}_d)^\top & N^{22}_d\end{bmatrix}$$

with $N^{22}>0$ and $N^{11}_d<0$.

Theorem 4: Given $\gamma > 0$, if there exist matrices $Y_s^i \in \mathbb{R}_s^{n_{xi}}, Y_k^i \in \mathbb{R}_s^{n_{xi}}, Z_s^i \in \mathbb{R}_s^{n_{xi}}, M_d^i \in \mathbb{R}_s^{n_{si}}, \quad M^i \in \mathbb{R}_s^{n_{si}}, S_d^i \in \mathbb{R}^{n_{si} \times n_{vi}}, S^i \in \mathbb{R}^{n_{si} \times n_{vi}}, R_d^i \in \mathbb{R}_s^{n_{vi}}$, and $R^i \in \mathbb{R}_s^{n_{vi}}$ $(i \in \Sigma)$ such that the LMI conditions (37)–(44) are feasible, then there exist controllers (26)–(27) such that the closed-loop system (28)–(29) is stable and the desired H_2 performance index is satisfied

$$(\star)^\top \underbrace{\begin{bmatrix} \mathrm{diag}\{\mathcal{M}_d^i|_{i=1}^N\} & \mathrm{diag}\{\mathcal{S}_d^i|_{i=1}^N\} \\ \mathrm{diag}\{(\mathcal{S}_d^i)^\top|_{i=1}^N\} & \mathrm{diag}\{\mathcal{R}_d^i|_{i=1}^N\} \end{bmatrix}}_{\mathcal{P}_d} \left[\begin{array}{cc} -\Psi^\top & \mathbf{0} \\ \mathbf{0} & -\Psi_k^\top \\ \hline I & \mathbf{0} \\ \mathbf{0} & I \end{array}\right] < 0 \tag{37}$$

$$M_d^i > M^i > 0 \tag{38}$$

$$R_d^i < R^i < 0 \tag{39}$$

$$\begin{bmatrix} Y_s^i & \mathbf{0} \\ \mathbf{0} & Y_k^i \end{bmatrix} > 0 \tag{40}$$

$$(\star)^\top \begin{bmatrix} Y_s^i & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & M^i & S^i & \mathbf{0} \\ \mathbf{0} & (S^i)^\top & R^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix} \begin{bmatrix} A_{xv}^i \\ A_{sv}^i \\ I^{n_{vi}} \\ C_{zv}^i \end{bmatrix}^{oc} > 0 \tag{41}$$

$$(\star\star)^\top W_i \begin{bmatrix} I & \mathbf{0} & \mathbf{0} \\ -(A_{xx}^i)^\top & -(A_{sx}^i)^\top & -(C_{zx}^i)^\top \\ \mathbf{0} & I & \mathbf{0} \\ -(A_{xv}^i)^\top & -(A_{sv}^i)^\top & -(C_{zv}^i)^\top \\ \mathbf{0} & \mathbf{0} & I \end{bmatrix} \begin{bmatrix} \mathcal{F}_1^i \\ \mathcal{F}_2^i \\ \mathcal{F}_3^i \end{bmatrix} > 0 \tag{42}$$

$$\begin{bmatrix} Z_s^i & I \\ I & Y_s^i \end{bmatrix} > 0 \tag{43}$$

$$\sum_{i=1}^N \mathrm{Trace}\left((B_{xw}^i)^\top Z_s^i B_{xw}^i \right) < \gamma^2 \tag{44}$$

where

$$W_i = \begin{bmatrix} Y_s^i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -Y_s^i & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & M^i{}_d & S_d^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (S_d^i)^\top & R_d^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix}$$

$$\mathcal{R}_d^i = \begin{bmatrix} R_d^i & R_d^i - R^i \\ R_d^i - R^i & R_d^i - R^i \end{bmatrix} \tag{45}$$

$$\mathcal{M}_d^i = \begin{bmatrix} M_d^i & M_d^i - M^i \\ M_d^i - M^i & M_d^i - M^i \end{bmatrix}, \mathcal{S}_d^i = \begin{bmatrix} S_d^i & S_d^i - S^i \\ S_d^i - S^i & S_d^i - S^i \end{bmatrix} \tag{46}$$

and $[(\mathcal{F}_1^i)^\top, (\mathcal{F}_2^i)^\top, (\mathcal{F}_3^i)^\top]^\top$ is a basis for the kernel of $[(B_{xu}^i)^\top, (B_{su}^i)^\top, (D_{zu}^i)^\top]$.

Proof: The proof will be completed through three parts. Part 1 shows that (37)–(39) $\Rightarrow$ (34). Part 2 shows that (38)–(42) $\Rightarrow$ (33). Part 3 shows that (43)–(44) $\Rightarrow$ (35). The above three parts implies that (37)–(44) $\Rightarrow$ (33)–(35). According to Theorem 3, the conditions (37)–(44) ensure the stability and H_2 performance of the closed-loop system (28)–(29).

Part 1: We first define the matrices J_1 and J_2 as

$$J_1 \triangleq \begin{bmatrix} \mathrm{diag}\{\mathrm{M}^{\mathrm{i}}|_{\mathrm{i}=1}^{\mathrm{N}}\} & \mathrm{diag}\{S^i|_{i=1}^N\} \\ \mathrm{diag}\{(S^i)^\top|_{i=1}^N\} & \mathrm{diag}\{R^i|_{i=1}^N\} \end{bmatrix} \tag{47}$$

$$J_2 \triangleq \begin{bmatrix} \mathrm{diag}\{\mathrm{M}_{\mathrm{d}}^{\mathrm{i}}|_{\mathrm{i}=1}^{\mathrm{N}}\} & \mathrm{diag}\{S_d^i|_{i=1}^N\} \\ \mathrm{diag}\{(S_d^i)^\top|_{i=1}^N\} & \mathrm{diag}\{R_d^i|_{i=1}^N\} \end{bmatrix}. \tag{48}$$

From (37) and (46)–(45), we have

$$\begin{aligned} \mathcal{P}_d &= \begin{bmatrix} \mathrm{diag}\{\mathcal{M}_{\mathrm{d}}^{\mathrm{i}}|_{\mathrm{i}=1}^{\mathrm{N}}\} & \mathrm{diag}\{\mathcal{S}_d^i|_{i=1}^N\} \\ \mathrm{diag}\{(\mathcal{S}_d^i)^\top|_{i=1}^N\} & \mathrm{diag}\{\mathcal{R}_d^i|_{i=1}^N\} \end{bmatrix} \\ &= \Pi_2^\top \left[\begin{array}{c|c} J_2 & J_2 - J_1 \\ \hline J_2 - J_1 & J_2 - J_1 \end{array}\right] \Pi_2 \end{aligned} \tag{49}$$

where Π_2 is given by

$$\Pi_2 = \left[\begin{array}{cccc|cccc} I & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & I & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & I & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & I & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & I & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & I \end{array}\right]^\top.$$

According to Lemma 3, we define that

$$\begin{aligned} J_1^{-1} &= \begin{bmatrix} \mathrm{diag}\{\mathrm{M}^{\mathrm{i}}|_{\mathrm{i}=1}^{\mathrm{N}}\} & \mathrm{diag}\{S^i|_{i=1}^N\} \\ \mathrm{diag}\{(S^i)^\top|_{i=1}^N\} & \mathrm{diag}\{R^i|_{i=1}^N\} \end{bmatrix}^{-1} \\ &\triangleq \begin{bmatrix} \mathrm{diag}\{\bar{\mathrm{M}}^{\mathrm{i}}|_{\mathrm{i}=1}^{\mathrm{N}}\} & \mathrm{diag}\{\bar{S}^i|_{i=1}^N\} \\ \mathrm{diag}\{(\bar{S}^i)^\top|_{i=1}^N\} & \mathrm{diag}\{\bar{R}^i|_{i=1}^N\} \end{bmatrix} \end{aligned}$$

$$\begin{aligned} (J_2 - J_1)^{-1} &= \begin{bmatrix} \mathrm{diag}\{(\mathrm{M}_{\mathrm{d}}^{\mathrm{i}} - \mathrm{M}^{\mathrm{i}})|_{\mathrm{i}=1}^{\mathrm{N}}\} & \mathrm{diag}\{(S_d^i - S^i)|_{i=1}^N\} \\ \mathrm{diag}\{(S_d^i - S^i)^\top|_{i=1}^N\} & \mathrm{diag}\{(R_d^i - R^i)|_{i=1}^N\} \end{bmatrix}^{-1} \\ &\triangleq \begin{bmatrix} \mathrm{diag}\{\mathrm{Q}_{\mathrm{s}}^{\mathrm{i}}|_{\mathrm{i}=1}^{\mathrm{N}}\} & \mathrm{diag}\{S_s^i|_{i=1}^N\} \\ \mathrm{diag}\{(S_s^i)^\top|_{i=1}^N\} & \mathrm{diag}\{R_s^i|_{i=1}^N\} \end{bmatrix}. \end{aligned} \tag{50}$$

According to the first property of Lemma 3, we have

$$\begin{aligned} &\mathrm{diag}\{\bar{M}^i|_{i=1}^N\} \\ &= \left\{\mathrm{diag}\left\{[M^i - S^i(R^i)^{-1}(S^i)^\top]|_{i=1}^N\right\}\right\}^{-1} > 0, \end{aligned} \tag{51}$$

$$\begin{aligned} &\mathrm{diag}\{Q_s^i|_{i=1}^N\} \\ &= \left\{\mathrm{diag}\{[(M_d^i - M^i) - (S_d^i - S^i)(R_d^i - R^i)^{-1}(S_d^i - S^i)^\top]|_{i=1}^N\}\right\}^{-1} \\ &> 0. \end{aligned} \tag{52}$$

Here, the ">" in (51) and (52) holds due to (38) and (39). We do not present the other matrices $\bar{S}^i, \bar{R}^i, S_s^i, R_s^i$ explicitly because they are not important for us. Although, they can also be computed according to Lemma 3.

Note that Π_2 is an orthogonal matrix, i.e., $\Pi_2\Pi_2^\top = \Pi_2^\top\Pi_2 = I$. We further obtain that

$$\begin{aligned}\mathcal{P}_d^{-1} &= \left(\Pi_2^\top \left[\begin{array}{c|c} J_2 & J_2-J_1 \\ \hline J_2-J_1 & J_2-J_1\end{array}\right]\Pi_2\right)^{-1} \\ &= \Pi_2^\top \left[\begin{array}{c|c} J_1^{-1} & -J_1^{-1} \\ \hline -J_1^{-1} & (J_2-J_1)^{-1}+J_1^{-1}\end{array}\right]\Pi_2 \\ &= \begin{bmatrix} \mathrm{diag}\{\mathcal{M}^i|_{i=1}^N\} & \mathrm{diag}\{\mathcal{S}^i|_{i=1}^N\} \\ \mathrm{diag}\{(\mathcal{S}^i)^\top|_{i=1}^N\} & \mathrm{diag}\{\mathcal{R}^i|_{i=1}^N\}\end{bmatrix} \end{aligned} \tag{53}$$

where

$$\mathcal{M}^i = \begin{bmatrix} \bar{M}^i & -\bar{M}^i \\ -\bar{M}^i & Q_s^i+\bar{M}^i\end{bmatrix}, \quad \mathcal{S}^i = \begin{bmatrix} \bar{S}^i & -\bar{S}^i \\ -\bar{S}^i & S_s^i+\bar{S}^i\end{bmatrix},$$

$$\mathcal{R}^i = \begin{bmatrix} \bar{R}^i & -\bar{R}^i \\ -\bar{R}^i & R_s^i+\bar{R}^i\end{bmatrix}. \tag{54}$$

The first "=" in (53) holds due to (49). The second "=" in (53) holds because of the matrix inversion formulas in [14, Sec. 2.3]. The last "=" in (53) holds according to direct mathematical manipulations.

Take into account (38)–(39), by utilizing Schur complement lemma to $\mathcal{M}_d^i$ and $\mathcal{R}_d^i$ in (46) and (45), respectively, we further obtain that

$$\mathrm{diag}\{\mathcal{M}_d^i|_{i=1}^N\} > 0, \quad \mathrm{diag}\{\mathcal{R}_d^i|_{i=1}^N\} < 0. \tag{55}$$

From the second property of Lemma 3, we have

$$\begin{aligned} &\mathrm{in}_-\left\{\begin{bmatrix} \mathrm{diag}\{\mathcal{M}_d^i|_{i=1}^N\} & \mathrm{diag}\{\mathcal{S}_d^i|_{i=1}^N\} \\ \mathrm{diag}\{(\mathcal{S}_d^i)^\top|_{i=1}^N\} & \mathrm{diag}\{\mathcal{R}_d^i|_{i=1}^N\}\end{bmatrix}\right\} \\ &= \mathrm{in}_-\{\mathrm{diag}\{\mathcal{R}_d^i|_{i=1}^N\}\} = \sum_{i=1}^N 2n_{vi}. \end{aligned} \tag{56}$$

On the other hand, because $\Phi \in \mathbb{R}^{S_v\times S_s}$ and $\Phi_k \in \mathbb{R}^{S_v\times S_s}$, we have

$$\dim\left\{\mathrm{im}\left\{\left[\begin{array}{cc} -\Psi^\top & \mathbf{0} \\ \mathbf{0} & -\Psi_k^\top \\ \hline I & \mathbf{0} \\ \mathbf{0} & I\end{array}\right]\right\}\right\} = \sum_{i=1}^N 2n_{vi}. \tag{57}$$

Having (56)–(57) in mind, applying Lemma 4 to (37) means that (34) is satisfied.

Part 2: The condition (33) can be rewritten as

$$\begin{aligned} (\star)^\top T^i N^i &= (\star\star)^\top T^i \Pi_3 \left[\begin{array}{cc} I & \mathbf{0} \\ \mathbf{0} & I \\ \hline (A_{xx}^i)_c & (A_{xv}^i)_c \\ (A_{sx}^i)_c & (A_{sv}^i)_c \\ (C_{zx}^i)_c & (C_{zv}^i)_c \end{array}\right] \\ &= (\star)^\top M^i \left[\begin{array}{c} I \\ \hline \mathcal{A}^i + (\mathcal{B}^i)^\top \mathcal{K}^i \mathcal{C}^i \end{array}\right] \\ &< 0, \quad i \in \Sigma \end{aligned} \tag{58}$$

where

$$\Pi_3 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & I & \mathbf{0} & \mathbf{0} \\ I & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix},$$

$$M^i = \Pi_3^\top T^i \Pi_3 = \left[\begin{array}{cc|ccc} -\mathcal{X}^i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathcal{R}^i & \mathbf{0} & (\mathcal{S}^i)^\top & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathcal{X}^i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathcal{S}^i & \mathbf{0} & \mathcal{M}^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{array}\right].$$

Define that

$$\mathcal{X}^i \triangleq \begin{bmatrix} Y_s^i & \mathbf{0} \\ \mathbf{0} & Y_k^i \end{bmatrix}^{-1} > 0, \quad i \in \Sigma. \tag{59}$$

Here, the ">" holds because of (40).

Taking into account (51) and (52), by utilizing Schur complement lemma to $\mathcal{M}^i$ in (54), we obtain $\mathcal{M}^i > 0$. Therefore

$$\begin{bmatrix} \mathcal{X}^i & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathcal{M}^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I \end{bmatrix} > 0, \quad i \in \Sigma. \tag{60}$$

From the structure of the matrix $\mathcal{P}_d$ given by (49) and $\mathcal{P}_d^{-1}$ given by (53), we have

$$\begin{bmatrix} \mathcal{M}^i & \mathcal{S}^i \\ (\mathcal{S}^i)^\top & \mathcal{R}^i \end{bmatrix} = \begin{bmatrix} \mathcal{M}_d^i & \mathcal{S}_d^i \\ (\mathcal{S}_d^i)^\top & \mathcal{R}_d^i \end{bmatrix}^{-1}.$$

Therefore

$$(T^i)^{-1} = \begin{bmatrix} (\mathcal{X}^i)^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -(\mathcal{X}^i)^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathcal{M}_d^i & \mathcal{S}_d^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (\mathcal{S}_d^i)^\top & \mathcal{R}_d^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix}. \tag{61}$$

Noting that Π_3 is an orthogonal matrix, we obtain that

$$\begin{aligned} (M^i)^{-1} &= \Pi_3^\top (T^i)^{-1}\Pi_3 \\ &= \left[\begin{array}{cc|ccc} -(\mathcal{X}^i)^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathcal{R}_d^i & \mathbf{0} & (\mathcal{S}_d^i)^\top & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & (\mathcal{X}^i)^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathcal{S}_d^i & \mathbf{0} & \mathcal{M}_d^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{array}\right]. \end{aligned} \tag{62}$$

Due to $\mathrm{diag}\{\mathcal{R}_d^i|_{i=1}^N\} < 0$ in (55), we obtain that

$$\begin{bmatrix} -(\mathcal{X}^i)^{-1} & \mathbf{0} \\ \mathbf{0} & \mathcal{R}_d^i \end{bmatrix} < 0, \quad i \in \Sigma. \tag{63}$$

Having (60) and (63) in mind, according to Lemma 5, (58) is feasible if and only if

$$(\star)^\top M^i \begin{bmatrix} \mathcal{C}_{bk}^i \\ \mathcal{A}^i\mathcal{C}_{bk}^i \end{bmatrix} < 0, \tag{64}$$

$$(\star)(M^i)^{-1} \begin{bmatrix} -(\mathcal{A}^i)^\top \mathcal{B}_{bk}^i \\ \mathcal{B}_{bk}^i \end{bmatrix} > 0 \tag{65}$$

hold for all $i \in \Sigma$. Here, $\mathcal{C}^i_{bk}$ and $\mathcal{B}^i_{bk}$ are used to represent a basis for the kernel of $\mathcal{C}^i$ and $\mathcal{B}^i$, respectively.

Now, we are going to proof that (64)–(65) are satisfied if (38)–(42) hold.

From the definitions of J_1 in (47) and J_1^{-1} in (50), we have

$$\begin{bmatrix} \bar{M}^i & \bar{S}^i \\ (\bar{S}^i)^\top & \bar{R}^i \end{bmatrix} = \begin{bmatrix} M^i & S^i \\ (S^i)^\top & R^i \end{bmatrix}^{-1}, \quad i \in \Sigma. \tag{66}$$

Therefore

$$\begin{aligned} \mathrm{in}_-\left\{\begin{bmatrix} \bar{M}^i & \bar{S}^i \\ (\bar{S}^i)^\top & \bar{R}^i \end{bmatrix}\right\} &= \mathrm{in}_-\left\{\begin{bmatrix} M^i & S^i \\ (S^i)^\top & R^i \end{bmatrix}^{-1}\right\} \\ &= \mathrm{in}_-\left\{\begin{bmatrix} M^i & S^i \\ (S^i)^\top & R^i \end{bmatrix}\right\} = \mathrm{in}_-\{R^i\} = n_{vi}. \end{aligned} \tag{67}$$

Here, the first "=" holds due to (66). The second "=" is satisfied because the negative eigenvalues number can not be changed by matrix inversion computation. The third "=" holds due to the second property of Lemma 3. The last "=" holds because of the fact that $R^i \in \mathbb{R}^{n_{vi}}_s$ and $R^i < 0$.

One further obtains that

$$\mathrm{in}_-\left\{\begin{bmatrix} (Y^i_s)^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{M}^i & \bar{S}^i & \mathbf{0} \\ \mathbf{0} & (\bar{S}^i)^\top & \bar{R}^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix}\right\} = \mathrm{in}_-\left\{\begin{bmatrix} \bar{M}^i & \bar{S}^i \\ (\bar{S}^i)^\top & \bar{R}^i \end{bmatrix}\right\} = n_{vi}. \tag{68}$$

Next, note the fact of linear independent columns, we get

$$\dim\left\{\mathrm{im}\left\{\begin{bmatrix} A^i_{xv} \\ A^i_{sv} \\ I^{n_{vi}} \\ C^i_{zv} \end{bmatrix}\right\}\right\} = n_{vi}. \tag{69}$$

The relations (68) and (69) are used for applying Lemma 4 as follows.

We can verify for (64) that

$$(\star)^\top M^i \begin{bmatrix} \mathcal{C}^i_{bk} \\ \mathcal{A}^i \mathcal{C}^i_{bk} \end{bmatrix} = (\star)^\top \begin{bmatrix} (Y^i_s)^{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \bar{M}^i & \bar{S}^i & \mathbf{0} \\ \mathbf{0} & (\bar{S}^i)^\top & \bar{R}^i & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I \end{bmatrix} \times \begin{bmatrix} A^i_{xv} \\ A^i_{sv} \\ I^{n_{vi}} \\ C^i_{zv} \end{bmatrix} < 0. \tag{70}$$

Here, $\mathcal{C}^i_{bk} = [\mathbf{0}\, \mathbf{0}\, I\, \mathbf{0}]^\top$ is a basis for the kernel of $\mathcal{C}^i$. The "<" in (70) is satisfied by applying Lemma 4 to (41).

Next, note the matrix $(T^i)^{-1}$ given by (61), the matrix $(M^i)^{-1} = \Pi_3^\top (T^i)^{-1} \Pi_3$ given by (62) and

$$\mathcal{B}^i_{bk} = \begin{bmatrix} \mathbf{0} & I & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & I & \mathbf{0} \\ (B^i_{xu})^\top & \mathbf{0} & (B^i_{su})^\top & \mathbf{0} & (D^i_{zu})^\top \end{bmatrix}_{bk} = \begin{bmatrix} \mathcal{F}^i_1 \\ \mathbf{0} \\ \mathcal{F}^i_2 \\ \mathbf{0} \\ \mathcal{F}^i_3 \end{bmatrix}.$$

One further obtains for (65) that

$$\begin{aligned} &(\star)^\top (M^i)^{-1} \begin{bmatrix} -(\mathcal{A}^i)^\top \mathcal{B}^i_{bk} \\ \mathcal{B}^i_{bk} \end{bmatrix} \\ &= (\star\star)^\top (T^i)^{-1} \left[\begin{array}{ccc} I & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline -(A^i_{xx})^\top & -(A^i_{sx})^\top & -(C^i_{zx})^\top \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline -(A^i_{xv})^\top & -(A^i_{sv})^\top & -(C^i_{zv})^\top \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & I \end{array}\right] \begin{bmatrix} \mathcal{F}^i_1 \\ \mathcal{F}^i_2 \\ \mathcal{F}^i_3 \end{bmatrix} \\ &= (\star\star)^\top W_i \begin{bmatrix} I & \mathbf{0} & \mathbf{0} \\ -(A^i_{xx})^\top & -(A^i_{sx})^\top & -(C^i_{zx})^\top \\ \mathbf{0} & I & \mathbf{0} \\ -(A^i_{xv})^\top & -(A^i_{sv})^\top & -(C^i_{zv})^\top \\ \mathbf{0} & \mathbf{0} & I \end{bmatrix} \begin{bmatrix} \mathcal{F}^i_1 \\ \mathcal{F}^i_2 \\ \mathcal{F}^i_3 \end{bmatrix} > 0. \end{aligned} \tag{71}$$

Here, the ">" in (71) holds due to (42).

From (70) and (71), we conclude that (58), which is rewritten as (33), is satisfied according to Lemma 5.

Part 3: Applying Schur complement lemma to (43) leads to

$$(Y^i_s)^{-1} < Z^i_s. \tag{72}$$

Note that $(B^i_{xw})_c = \begin{bmatrix} B^i_{xw} \\ \mathbf{0} \end{bmatrix}$ and $\mathcal{X}^i = \begin{bmatrix} (Y^i_s)^{-1} & \mathbf{0} \\ \mathbf{0} & (Y^i_k)^{-1} \end{bmatrix}$ given by (31) and (59), respectively, we obtain for (35) that

$$\begin{aligned} &\sum_{i=1}^N \mathrm{Trace}((B^i_{xw})_c^\top \mathcal{X}^i (B^i_{xw})_c) \\ &= \sum_{i=1}^N \mathrm{Trace}\left(\begin{bmatrix} B^i_{xw} \\ \mathbf{0} \end{bmatrix}^\top \begin{bmatrix} (Y^i_s)^{-1} & \mathbf{0} \\ \mathbf{0} & (Y^i_k)^{-1} \end{bmatrix} \begin{bmatrix} B^i_{xw} \\ \mathbf{0} \end{bmatrix}\right) \\ &= \sum_{i=1}^N \mathrm{Trace}\left((B^i_{xw})^\top (Y^i_s)^{-1} B^i_{xw}\right) \\ &< \sum_{i=1}^N \mathrm{Trace}\left((B^i_{xw})^\top Z^i_s B^i_{xw}\right) < \gamma^2. \end{aligned}$$

Here, the first "<" holds due to (72); the last "<" holds because of (44). ■

Remark 6: The minimum value $\gamma_{\min}$ is of great significance in some practical engineering examples, which can be obtained according to the following optimization problem:

$$\min_{\substack{Y_s^i\in\mathbb{R}_s^{n_{xi}},Y_k^i\in\mathbb{R}_s^{n_{xi}},Z_s^i\in\mathbb{R}_s^{n_{xi}},M_d^i\in\mathbb{R}_s^{n_{si}},\\ M^i\in\mathbb{R}_s^{n_{si}},S_d^i\in\mathbb{R}^{n_{si}\times n_{vi}},S^i\in\mathbb{R}^{n_{si}\times n_{vi}},\\ R_d^i\in\mathbb{R}_s^{n_{vi}},R^i\in\mathbb{R}_s^{n_{vi}}}} \gamma^2 \quad \text{subject to} \quad (37)\text{–}(44). \tag{73}$$

B. Controller Construction

If the LMI conditions (37)–(44) are solvable, the inequality (33), which is rewritten as (58), is also satisfied. According to [8, Lemma 1], all the controller gains in $\mathcal{K}^i$ ($i\in\Sigma$) are constructed by the following steps.

1) Compute $M^i=\Pi_3^\top T^i\Pi_3$ and

$$\begin{bmatrix}\mathcal{M}^i & \mathcal{S}^i\\ (\mathcal{S}^i)^\top & \mathcal{R}^i\end{bmatrix}=\begin{bmatrix}\mathcal{M}_d^i & \mathcal{S}_d^i\\ (\mathcal{S}_d^i)^\top & \mathcal{R}_d^i\end{bmatrix}^{-1},\quad \mathcal{X}^i=\begin{bmatrix}Y_s^i & \mathbf{0}\\ \mathbf{0} & Y_k^i\end{bmatrix}^{-1}.$$

2) Introduce two matrices H^i and U^i which are nonsingular such that

$$\mathcal{B}^iH^i=[\bar{\mathcal{B}}^i\ \mathbf{0}],\quad \mathcal{C}^iU^i=[\bar{\mathcal{C}}^i\ \mathbf{0}]$$

where $\bar{\mathcal{C}}^i$ and $\bar{\mathcal{B}}^i$ are of full column rank.

3) Define that

$$Q^i\triangleq\begin{bmatrix}I & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\\ \mathbf{0} & I\\ F_{21}^i & \mathbf{0}\end{bmatrix},\ S^i\triangleq\begin{bmatrix}\mathbf{0}\\ I\\ F_{12}^i\\ F_{22}^i\end{bmatrix},$$

$$N^i\triangleq(\star)^\top M^i\begin{bmatrix}U^i & \mathbf{0}\\ \mathbf{0} & ((H^i)^\top)^{-1}\end{bmatrix}$$

where the matrix $F^i=(H^i)^\top\mathcal{A}^iU^i\triangleq\begin{bmatrix}F_{11}^i & F_{12}^i\\ F_{21}^i & F_{22}^i\end{bmatrix}$. Here, the size of F_{11}^i is the same as $(\bar{\mathcal{B}}^i)^\top\mathcal{K}^i\bar{\mathcal{C}}^i$.

4) A satisfactory $\mathcal{K}^i$ is given by

$$\mathcal{K}^i=\bar{\mathcal{B}}^i((\bar{\mathcal{B}}^i)^\top\bar{\mathcal{B}}^i)^{-1}(Z^i-F_{11}^i)((\bar{\mathcal{C}}^i)^\top\bar{\mathcal{C}}^i)^{-1}(\bar{\mathcal{C}}^i)^\top$$

where $Z^i=Z_2^i(Z_1^i)^{-1}$ and Z_1^i,Z_2^i satisfy

$$(\star)^\top\left((Q^i)^\top N^iQ^i-(Q^i)^\top N^iS^i((S^i)^\top N^i\right.$$
$$\left.\times(S^i))^{-1}(S^i)^\top N^iQ^i\right)\begin{bmatrix}Z_1^i\\ Z_2^i\end{bmatrix}<0. \tag{74}$$

Remark 7: By diagonalizing the matrix $(Q^i)^\top N^iQ^i-(Q^i)^\top N^iS^i((S^i)^\top N^i(S^i))^{-1}(S^i)^\top N^iQ^i$ in an orthogonal basis, the columns of $\begin{bmatrix}Z_1^i\\ Z_2^i\end{bmatrix}$ in (74) are elected as the eigenvectors associated with the negative eigenvalues.

Remark 8: It should be mentioned that the feasibility of the LMI conditions in Theorem 4 require some global information. However, once the LMIs are solved and the solutions are used to construct the controller parameters based on the four steps in Section IV-B, the implementation of the controllers are distributed. That is, each subcontroller only requires local subsystem and its neighboring subsystem's information to form its control input. In our existing work [21], similar techniques are also used to design the controller gains. However, the authors of [21] studied the H_∞ control for LSSs, while this article mainly considers H_2 performance analysis and designs distributed H_2 controllers of interconnected LSSs.

Remark 9: Note that in the four steps of the controller construction in Section IV-B, there are some matrix inversions "$((H^i)^\top)^{-1},((\bar{\mathcal{B}}^i)^\top\bar{\mathcal{B}}^i)^{-1},((\bar{\mathcal{C}}^i)^\top\bar{\mathcal{C}}^i)^{-1},Z_2^i(Z_1^i)^{-1}$" needed to be computed. However, the dimension of matrix inversion computation does not increase with the number of subsystems. The dimension of each matrix inversion term is only related to its subsystem's system matrices. Compared with the method in Lemma 1, [12] and [15], the dimension of the matrix inversions increases with the number of subsystems, the computation of matrix inversions are computationally expensive and may even cause numerically unstable problems especially for a large number of subsystems. The advantage of our controller construction method is that the large dimensional matrix inversion terms have been eliminated.

Remark 10: In this article, Theorems 1 and 2 are proposed for open-loop LSSs. Theorems 3 and 4 are proposed for closed-loop ILSSs. Theorem 1 proposes both novel necessary and sufficient conditions to ensure the stability and the prescribed H_2 performance for the considered open-loop LSS. The contribution of Theorem 1 is that compared with the existing method in Lemma 1, the derived conditions in Theorem 1 are computationally attractive because of the elimination of the computationally expensive matrix inversion terms. Theorem 2 proposes sufficient conditions to ensure the stability and the prescribed H_2 performance for open-loop LSSs. The conditions in Theorem 2 are also free of computationally expensive matrix inversion terms. Another advantage of Theorem 2 is that compared with the conditions in Theorem 1, the size of LMI conditions to be solved and the number of decision variables are less than those in Theorem 1 (the comparison results can be found in Table I). Theorem 3 is the direct extension of Theorem 2, where the derived conditions ensure the stability and H_2 performance for the considered closed-loop LSS. Theorem 4 has transformed the sufficient nonlinear conditions in Theorem 3 into LMI conditions, which are easy to solve. Finally, the solutions of the LMI conditions in Theorem 4 can be used to construct the distributed H_2 controller parameters.

V. Examples

Three examples are presented in this section to illustrate the superiorities and advantages of our developed control scheme.

Example 1: This example is utilized to compare our developed controller design approach to the approach in [12] in the situation of different system parameters.

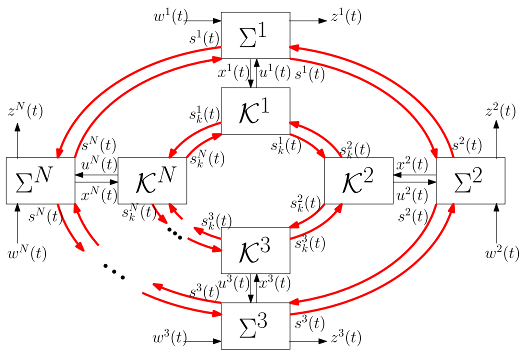

Fig. 2. System structure of an interconnected LSS which is connected to a ring.

Consider an LSS with N subsystems in Fig. 2. Here, we consider that the distributed subcontrollers share the same interconnection relation as subsystems, i.e.,

$$v^1(t) = \begin{bmatrix} s^2(t) \\ s^N(t) \end{bmatrix}, \; v^i(t) = \begin{bmatrix} s^{i-1}(t) \\ s^{i+1}(t) \end{bmatrix}, \; v^N(t) = \begin{bmatrix} s^{N-1}(t) \\ s^1(t) \end{bmatrix},$$

$$v_k^1(t) = \begin{bmatrix} s_k^2(t) \\ s_k^N(t) \end{bmatrix}, \; v_k^i(t) = \begin{bmatrix} s_k^{i-1}(t) \\ s_k^{i+1}(t) \end{bmatrix}, \; v_k^N(t) = \begin{bmatrix} s_k^{N-1}(t) \\ s_k^1(t) \end{bmatrix}$$

where $i \in \{2, 3, \ldots, N-1\}$.

The state space representation of the ith ($i \in \{1, 2, \ldots, N\}$) subsystem is given by (1). We aim to compare our approach with the approach in [12], let $A_{sv}^i, B_{su}^i, C_{zv}^i, D_{zu}^i$ ($i \in \{1, 2, \ldots, N\}$) be zero matrices and

$$\underline{M} = \begin{bmatrix} 0.3 & 0.1a \\ 0 & 0.1 \end{bmatrix}, \overline{M} = \begin{bmatrix} 0.1 & 0.1b \\ 0 & 0.4+0.1a \end{bmatrix}, B_{xw}^i = \begin{bmatrix} a-0.5 \\ b \end{bmatrix},$$

$$A_{xx}^i = \underline{M} + 2\overline{M}, \; A_{xv}^i = \begin{bmatrix} -\overline{M} & -\overline{M} \end{bmatrix},$$

$$B_{xu}^i = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad C_{zx}^i = [0.3 \; 0], \quad A_{sx}^i = I.$$

Here, "a" and "b" are parameterized scalars.

The considered system can be verified to be a "decomposable system" [12] when the "pattern matrix" in [12] is given by the Laplacian matrix of the interconnection graph of Fig. 2. Thus, the results in [12] can be applied to this example.

Next, considering the following three situations, we make a comparison of the value $\gamma_{\min}$ achieved by our approach with the value achieved by the approach in [12].

1) $N = 10$, $b = 0$, $a \in \{0.1, 0.2, \ldots, 1\}$;
2) $N = 10$, $a = 0$, $b \in \{0.1, 0.2, \ldots, 1\}$;
3) $a = 0.3$, $b = 0.2$, $N \in \{10, 20, \ldots, 100\}$.

Based on the optimization problem (73) in Remark 6 and [12, Th. 15], the comparison results are given in Figs. 3, 4, and 5.

It follows from Figs. 3 and 4 that in the case of fixed number of subsystems $N = 10$, the minimal value $\gamma_{\min}$ achieved by our approach is smaller than that by the approach in [12] in the case of various system parameterized scalars "a" and "b." On the other hand, Fig. 5 shows that if we fix $a = 0.3$ and $b = 0.2$, the minimal value $\gamma_{\min}$ obtained by our approach is also smaller than that by the approach in [12] in the case of different number of subsystems.

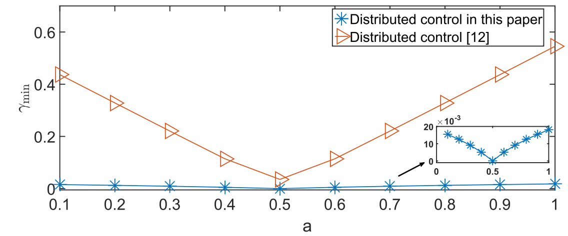

Fig. 3. Value of $\gamma_{\min}$ achieved by our method and the method in [12] under different "a" ($b = 0$, $N = 10$).

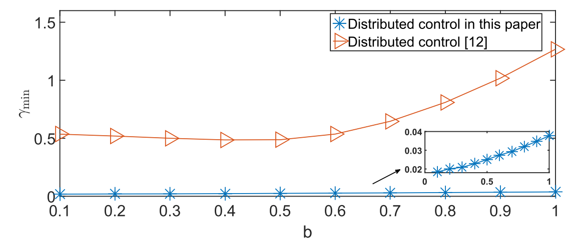

Fig. 4. Value of $\gamma_{\min}$ achieved by our method and the method in [12] under different "b" ($a = 0$, $N = 10$).

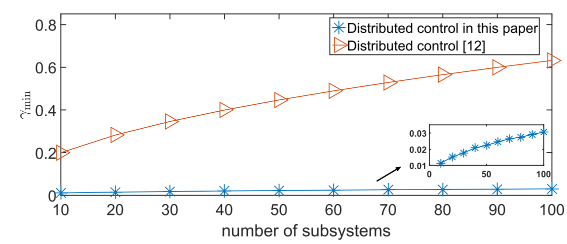

Fig. 5. Value of $\gamma_{\min}$ achieved by our method and the method in [12] under different "N" ($a = 0.3$, $b = 0.2$).

Therefore, compared with the method in [12], our method is less conservative, better closed-loop system H_2 performance is obtained by our distributed controller design approach.

Example 2: This example studies an LSS, which consists of $N = 500$ subsystems. Here, the interconnection relation of subcontrollers is different from the interconnection relation of subsystems; see Fig. 6. The subsystems' interconnection and the subcontrollers' interconnection are

$$\begin{cases} v^1(t) = s^{500}(t); \\ v^i(t) = s^{i-1}(t); \\ v^{500}(t) = s^{499}(t); \end{cases} \qquad \begin{cases} v_k^1(t) = s_k^2(t); \\ v_k^i(t) = s_k^{i+1}(t); \\ v_k^{500}(t) = s_k^1(t) \end{cases}$$

where $i \in \{2, 3, \ldots, 499\}$.

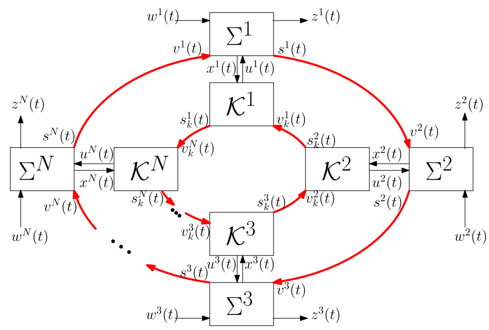

Fig. 6. System structure of an interconnected LSS with different subsystems's interconnection relation in Fig. 2.

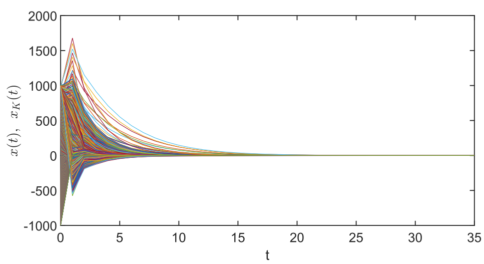

Fig. 7. State trajectories of the closed-loop LSS.

The system parameters in (1) are given by

$$A_{xx}^i=\begin{cases}\begin{bmatrix}\frac{3+\mathrm{mod}(i,5)}{10} & \frac{\mathrm{mod}(i,8)}{5}\\ 0 & \frac{12+\mathrm{mod}(i,7)}{10}\end{bmatrix}, & \text{if } i{=}3m{-}1, m\in\mathbb{N}^+;\\ \begin{bmatrix}\frac{1+\mathrm{mod}(i,4)}{5} & \frac{i}{2i+1}\\ 0 & \frac{\mathrm{mod}(i,7)}{10}\end{bmatrix}, & \text{otherwise,}\end{cases}$$

$$A_{xv}^i=\begin{bmatrix}\frac{\mathrm{mod}(i,3)}{5} & \frac{\mathrm{mod}(i,4)}{10}\\ 0 & 0.1\end{bmatrix},\quad B_{xw}^i=\begin{bmatrix}0.5\\ \frac{\mathrm{mod}(i,3)}{5}\end{bmatrix},$$

$$B_{xu}^i=\begin{bmatrix}\frac{\mathrm{mod}(i,2)}{5}\\ 0.5\end{bmatrix},\quad C_{zx}^i=\begin{bmatrix}0.3 & \frac{\mathrm{mod}(i,2)}{10}\end{bmatrix}.$$

The other matrices are given in Example 1. Here, "$\mathrm{mod}(m,n)$" is used to denote the remainder when "m" is divided by "n."

It is worth noting that in the situation of $i=3m-1$ ($m\in\mathbb{N}^+$), some eigenvalues of A_{xx}^i locate outside the unit circle of the complex plane. Also, one can verify that 16.5% eigenvalues of $A(\Psi)=A_{xx}+A_{xv}(I-\Psi A_{sv})^{-1}\Psi A_{sx}$ are outside the unit circle of the complex plane. That is, the considered LSS is open-loop unstable.

It is worthwhile to point out that this LSS is not a "decomposable system" [12]. One cannot apply the results in [12] for the controller design issues.

By solving the optimization problem (73), we obtained that $\gamma_{\min}$=0.0274 and the distributed controller parameters $\mathcal{K}^i$ ($i=1,2,\ldots,500$). However, the controller parameters are not presented explicitly here because of space limitation. If we set $x^i(0)=[2i-1,2i]^\top$ and $x_k^i(0)=[1-2i,-2i]^\top$ ($i=1,2,\ldots,500$), the state trajectories of the closed-loop system are given in Fig. 7, which has conformed that the controlled LSS is stable.

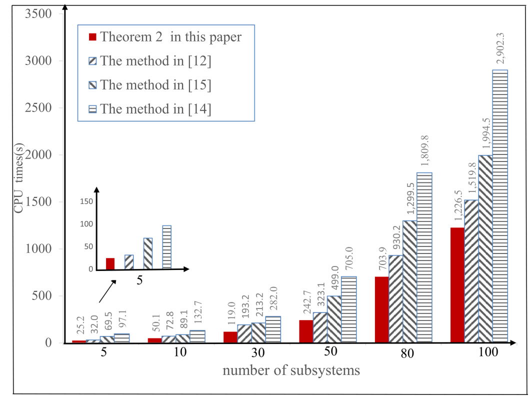

Fig. 8. CPU computation time of different methods for different number of subsystems.

Example 3: This example is executed to compare the computational time cost by different controller design methods under different number of subsystems, i.e., $N\in\{5,10,30,50,80,100\}$.

The interconnection relationships of different subsystems of the studied LSS are also considered in Fig. 2. The dynamics of the studied LSS are given by (1) with the system matrix parameters

$$\underline{M}=\begin{bmatrix}0.4 & -0.1\\ 0 & -0.3\end{bmatrix},\ \overline{M}=\begin{bmatrix}0.1 & 0\\ 0 & 0.3\end{bmatrix},\ B_{xw}^i=\begin{bmatrix}0.7\\ 0.2\end{bmatrix},$$

$$A_{xx}^i=\underline{M}+2\overline{M},\ A_{xv}^i=\begin{bmatrix}-\overline{M} & -\overline{M}\end{bmatrix},\ A_{sv}=\begin{bmatrix}0.1 & 0\\ 0 & 0.2\end{bmatrix},$$

$$B_{xu}^i=\begin{bmatrix}0 & 0.5\end{bmatrix}^\top,\quad C_{zx}^i=[0.1\ 0],\quad A_{sx}^i=I.$$

Now, let us design the controllers by our method and the methods in [12], [14] (Lemma 1 in this article), and [15], respectively. The controller design procedure is executed by our personal computer software MATLAB (Inter(R) 46 Core(TM) i5-2400 CPU). The simulation results in Fig. 8 compares the computation time of different controller design methods under the case of different number of subsystems.

From Fig. 8, we can see that under different number of subsystems, the computational time cost by our developed method is always less than those methods in [12], [14] (Lemma 1 in this article), and [15], which means that our developed controller design method is more computationally attractive. That is, the established controller design method in this article is more applicable and convincing especially for LSSs with a large number of subsystems.

VI. Conclusion

In this article, we have investigated the H_2 performance analysis and the distributed H_2 controller design problems for LSSs. By the developed space construction method, we have investigated both necessary and sufficient conditions to guarantee the stability and the H_2 performance of LSSs. Based on these results, sufficient LMI conditions are provided for the existence of the H_2 distributed controllers. All the controller gains are constructed via the solutions of the developed LMI conditions. Finally, three examples are introduced to show the validity and superiority of the proposed results of this article.

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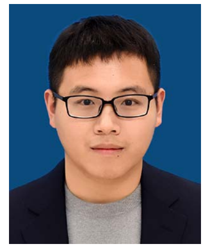

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