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ORIGINAL RESEARCH

Time-sensitive coverage control for non-holonomic and heterogeneous robots: An extremum seeking framework and application

Liang Zhang | Jinghui Deng | Kun Zhou | Tao Yu | Jun Song | Shuping He

Key Laboratory of Intelligent Computing and Signal Processing (Ministry of Education of the People's Republic of China), School of Electrical Engineering and Automation, Anhui University, Hefei, P. R. China

Correspondence
Shuping He, Key Laboratory of Intelligent Computing and Signal Processing (Ministry of Education of the People's Republic of China), School of Electrical Engineering and Automation, Anhui University, Hefei, 230601, P. R. China.
Email: shuping.he@ahu.edu.cn

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Abstract

This work addresses a time-sensitive coverage control problem for multiple non-holonomic mobile robots. The coverage objective is evaluated by the total arrivial time of robots reaching their dominant clients with the predefined constant but heterogeneous velocities. An extremum-seeking control framework is proposed, including a numerical optimizer that iteratively produces the optimal waypoints of robots minimizing the total arriving time and a state regulator that drives the robot's positions to the optimal locations under both constraints non-holonomic kinematic model and the constant-velocity. An improved K-means algorithm is first designed to produce the waypoint sequence. A theoretical analysis of the convergence of coverage objectives is provided. Then a fixed-time state regulator for the robot's angular velocity is derived from regulating robot positions to the desired waypoints within each time step that varies with the heterogeneous traversing velocity and the distance between waypoints. Finally, the results are validated by a virtual robotic simulation platform and the experiment on real robots, showing that the proposed methods can efficiently deploy multiple heterogeneous robots for time-sensitive services to clients with a discrete distribution. The simulation and experimental code are open-sourced at https://gitee.com/ahu-icip/simulation.git.

1 | INTRODUCTION

Recent years have witnessed the rapid development of coverage control as one of the most important research in multi-robots because of its wide application in military, research and rescue mission, environmental monitoring, space exploration, and so on [1–3]. The basic meaning of coverage control is that the robots could be well-deployed in their dominant region given the detected distribution of clients, such that robots could efficiently provide their services to the clients according to some predefined metrics (e.g., Euclidean distance).

Early works attempted solving the coverage control by the minimization of a locational cost indicating how well the domain is covered, where a continuous-time variation of Lloyd's algorithm based on gradient descent updating law was proposed in [4]. Hereafter, more techniques were developed, such as detection probability, Voronoi diagram [3], energy consumption [5] and localization [6]. Specifically, inspired by the method of central Voronoi partitions [7] and the location optimization problem in literature [8], Cortes et al. [4] defined a new problem of coverage control for multi-vehicle systems. It designed a distributed control law to optimize the position of multiple vehicles to achieve sensing measurement of the task area. After that, based on the gradient descent of the cost function and the Voronoi partitions, many researchers have further studied the coverage control problem of mobile agent networks and considered the physical constraints of mobile agents such as speed limit [9, 10], limited sensing or communication range [11], information density [12] in the task area, and so on [13]. In [14], the authors studied the effect of measurement error on the coverage cost function. The coverage cost function was defined as the maximum time for mobile sensors network to any point on the circle. In [15], the author proposed a distributed control method based on Voronoi tessellation and Delaunay graph-based local information sharing to achieve continuous coverage performance of multi-agent system. In [16], a distributed control law is proposed to solve the coverage control problem for mobile sensor networks with limited communication ranges on a circle. Literature [17] studies the problem of uniform coverage control when the number of neighbours of each mobile agent is limited. Lekien and Leonard [18] proposed a coverage control law using a statistical histogram transformation and solved the problem of non-uniform coverage control in a plane with a non-Euclidean distance measure. Literature [19] studied the coverage control of a group of mobile agents in intermittent communication over the non-uniform plane.

In many practical scenarios of covering tasks, the multi-robot system is usually heterogeneous due to the different motion mechanisms, which leads to the practical idea in coverage control problem that a faster robot is responsible for a larger area than a slower one in [9, 20]. They resize the Voronoi cells (the area the robot is responsible for) through an additively weighted Voronoi diagram. More coverage control works for heterogeneous robots can be found since then. In [21], the authors introduced a novel formulation for sensor coverage by multi-robot teams with heterogeneous sensing capabilities that maximize each robot's sensing quality, balancing the varying sensing capabilities of individual robots based on the overall team composition. [22] considered the coverage of different terrain types by different mobility types of robots, and each robot has its corresponding specific terrain type. In [23–25], the scholars studied the problem of multiple event types region effectively monitored by the robots detecting specific events. The coverage control for mobile sensor networks with limited communication ranges was addressed in [16], where a distributed control scheme with input saturation is developed to drive the sensors to the optimal configuration without network connectivity of the sensors. Other variations on these topics include energy consumption considerations.

Most previous works only considered the maximization of coverage area with spatial service or the minimization of coverage energy consumption [5, 10, 26, 27]. However, the coverage deployment of robots with temporal service is also very important for many practical applications, such as search and rescue tasks. The main objective is to act as soon as possible and provide services, that is, to minimize the time for a group of mobile agents to reach each point in the task area. For example, let us consider deploying ambulances in a residential area. It is not to detect where the patient is, but to get the ambulance to the patient's location as quickly as possible. There are two main stages. The ambulances (robots) must distribute themselves effectively in the first stage. In the second stage, when a patient is detected, the ambulance is driven to the patient at maximum speed and provides first-aid treatment. As considered in [28], the deployment problem of robots with temporal services is called time-sensitive coverage control in this paper. In this work, a coverage control method is proposed based on multiplicatively weighted Voronoi diagrams for robots with different maximum speeds in continuous distribution which minimizes the time for each agent to reach any points in its region of dominance.

However, [28] did not consider how to solve the time-sensitive coverage control problem for a heterogeneous multiple-robot system with a discrete distribution. The K-means algorithm proposed in [29] can solve the problem of discrete distribution for clients. It is an important analysis method in data mining, whose goal is to divide the data set into several clusters, making the clients within the same cluster similarity as large as possible and the clients of similarity between different clusters as small as possible. We can regard the clients within the same cluster as a whole as being covered by a robot. Therefore, the coverage control problem is combined with the K-means algorithm to obtain a new coverage control method to solve the coverage problem that is difficult to be solved by the previous coverage control methods. The authors in [30] proposed a novel coverage control method based on K-means, which can cover non-convex specific areas and discrete clients. They designed a dynamic coverage control law based on discrete sliding mode control to drive the mobile agents to the optimal coverage position. In [31], researchers studied coverage control for heterogeneous mobile sensor networks on a circle, which deployed five mobile sensors with different maximum velocities. They solved time-based coverage control on a circle, and mobile sensors keep their spatial position order in position evolution.

Based on the abovementioned discussion, this paper considers the time-sensitive coverage control for non-holonomic and heterogeneous robots for clients with discrete distributions. We utilize the K-means algorithm to complete the first stagethat is, effectively deploying the robots around the mission space by minimizing a cost function based on the minimal arriving time. Subsequently, the corresponding controllers are designed to drive the robots to the optimal locations that minimize the cost function. Moreover, the deployment achieves effective coverage based on the minimal arrival time rather than the euclidean distancethat is, the time-sensitive coverage control problem is solved.

The main contributions are as follows:

(1) In this paper, the time-sensitive coverage control of non-holonomic heterogeneous robots with constant speed is considered, and an extremum-seeking control framework is proposed for the time-sensitive coverage control problem such that its solution can be uniformly derived by a numerical optimizer producing the local optimal robot locations and a state regulator driving the robots to its desired deployment.
(2) We extend the *K-means* clustering algorithm as presented in [30] to the time-based coverage services of clients with a discrete distribution. Strict proof of its convergence is provided. Because of the non-holonomic mobile robots with constant-velocity constraints, a novel fixed-time control law is proposed.

(3) Results from both simulation and real-platform experiments are presented to validate the correctness and effectiveness of the proposed methods. Simulation and experimental code have been open-sourced.

2 | PROBLEM FORMULATION

Consider a network of m mobile robots moving in a two-dimensional mission space $\boldsymbol{F} \in \mathbb{R}^2$. Denote $\mathcal{P} = \{\boldsymbol{p}_1, \boldsymbol{p}_2, \dots, \boldsymbol{p}_m\}$ the set of all robots, where $\boldsymbol{p}_i \in \boldsymbol{F}$ is the position of ith robot. In contrast to the previous work, a non-holonomic kinematic model is considered for all robots,

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \tag{1}$$

where x_i and y_i are coordinates of the ith mobile sensor in the two-dimensional plane. θ_i is the angle between the velocity direction of the ith mobile sensor and the positive x-axis. v_i and ω_i represent the robot's linear and angular velocities, respectively. Given $\boldsymbol{\lambda} = \{\lambda_1, \lambda_2, \dots, \lambda_m\}$ a set of positive constants, in the following, we explicitly consider the constant-velocity constraint for all robots that is

$$v_i \equiv \lambda_i, \forall \boldsymbol{p}_i \in \mathcal{P} \tag{2}$$

There are simultaneously n clients around the space, denoted by $\mathcal{D} = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_n\}$, where $\boldsymbol{x}_j \in \boldsymbol{F}$ is the position of the jth client. The robots are dedicated to providing some time-sensitive services to clients, for example, a mobile fire truck to provide fire-fighting service or an ambulance for emergency medical service. Therefore, the quality of service of each client is mainly evaluated by the minimal arriving time of robots that is

$$t_j(\mathcal{P}) = \min_{\boldsymbol{p}_i \in \mathcal{P}} \frac{||\boldsymbol{x}_j - \boldsymbol{p}_i||_2}{\lambda_i} \tag{3}$$

Without loss of generality, we assume $n >> m$ implying each robot is responsible for multiple clients. Therefore, the overall objective function is defined as

$$H(\mathcal{P}) = \sum_{\boldsymbol{x}_j \in D} t_j \tag{4}$$

Inspired by the extremum seeking control strategy as proposed in [32, 33], the framework of the time-sensitive coverage control problem considered in this paper can be formulated as follows:

- **Step 0**: Given the initial robot positions $\mathcal{P}^0$, set $t_0 = 0$ and $k = 0$.
- **Step 1**: According to the known clients distribution $\mathcal{D}$, producing the local optimal location of robots $\mathcal{P}^{k+1}$ such that

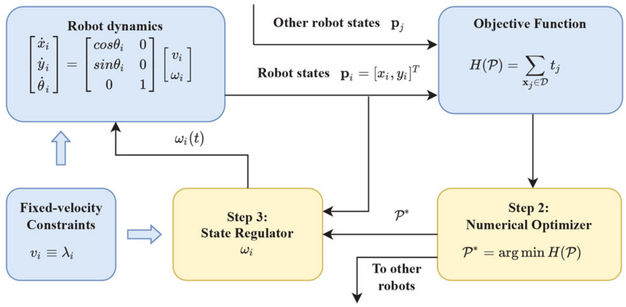


FIGURE 1 The structure of the control law of the time-sensitive coverage control problem.

the objective function achieves its minimum,

$$\mathcal{P}^{k+1} = \text{OPTIMIZER}\big(H(\mathcal{P}^k)\big)$$

- **Step 2**: Designing a state regulator w_i for all robots that regulates the position $\mathcal{P}(t)$ to $\mathcal{P}^{k+1}$ under the constraints of (1)–(2) in a finite time δ_k, let $t_{k+1} = t_k + \delta_k$, which implies

$$\mathcal{P}(t_{k+1}) = \mathcal{P}^{k+1}.$$

  where δ_k is a time-varying time step determined by the distance between consecutive waypoints and the constant traversing velocity of robot.
- **Step 3**: Set $k \leftarrow k + 1$, go to Step 1.

Remark that the underlying problem is similar to [28], where mobile robots are deployed to monitor a continuous mission space with different maximum speeds. In contrast, this work explicitly focuses on the discrete distribution of clients and the robots are all subject to the constant-velocity constraint. Therefore only the angular velocity w_i can be controlled. The under-actuation is one of the major challenges in optimizing the aforementioned problem. It is necessary to consider the constant-velocity constraint to guarantee the punctuality of time-sensitive services for every client.

Besides, the time-sensitive coverage control problem is mainly for multi-robot systems with different mobility. The problem considers how to deploy robots that can detect events and rapidly arrive at the events. For example, the purpose of a fire engine is not only to detect a fire but to know where there is a fire and arrive as soon as possible to extinguish it. Therefore, the faster robot should be responsible for more clients than the slower one. Finally, we designed the overall objective function based on time instead of distance. The time-sensitive coverage control problem is the overall objective, while the fixed-time control is a method to regulate robots reaching desired way-points. Specifically, the time-sensitive coverage control includes two parts: optimizing the temporally locational cost to produce optimal deployment of robots and then controlling the robot to reach the optimal deployment. The first part is determining the optimal coverage positions, where the K-means algorithm is applied. The second part, that is, the fixed-time controller, aims to make the robot quickly reach the optimal coverage position given the constant-velocity constraint.

Lastly, the extremum-seeking control framework, as shown in Figure 1, is an optimal control approach that deals with situations when the plant model and the cost to optimize are not available to the designer. It is useful for adapting parameters to unknown system dynamics and unknown mappings from control parameters to an objective function. Therefore, it is suitable for the time-sensitive coverage problem where the temporally locational cost should be minimized (or maximized).

3 | MAIN RESULTS

The overall problem includes two main parts, the numerical optimizer and the state regulator, respectively. Firstly, we combine the K-means algorithm and coverage control problem to design a numerical optimizer to produce the local optimum of robot positions. We also provide strict proof of the convergence and optimality of the objective in (9). We then design a state regulator incorporating the fixed-time control law regarding non-holonomic dynamics and constant velocity constraints.

3.1 | Numerical optimizer for the time-sensitive coverage control problem

This subsection details the numerical optimizer for discrete clients' time-sensitive coverage control problem given the initial position. Given the initial position of robots $\mathcal{P}^0$, we firstly define the dominance of ith robot as

$$C_i^k = \left\{ \forall \boldsymbol{x}_j \in \mathcal{D} \middle| \frac{||\boldsymbol{x}_j - \boldsymbol{p}_i||}{\lambda_i} \leq \frac{||\boldsymbol{x}_j - \boldsymbol{p}_k||}{\lambda_k}, \forall \boldsymbol{p}_k \in \mathcal{P}^k \right\} \tag{5}$$

$\forall k \in \{0, 1, 2, \ldots\}$, where k is the index of updating iteration.

Note that according to the definition, the corresponding robot of the dominance set C_i is the fastest node that can provide the time-sensitive service to all clients that are included in the set. Therefore, the client space $\mathcal{D}$ is clustered by the overall set $C = \{C_1, C_2, \ldots, C_m\}$ according to the fastest arrival time. Then we can define the centres of each cluster in C as $\bar{\mathcal{P}}^k = \{\bar{\boldsymbol{p}}_1^k, \bar{\boldsymbol{p}}_2^k, \ldots, \bar{\boldsymbol{p}}_m^k\}$ by

$$\bar{\boldsymbol{p}}_i^{k+1} = \frac{1}{N_{C_i^k}} \sum_{j \in C_i^k} \boldsymbol{x}_j, \forall i \in \{1, 2, \ldots, m\} \tag{6}$$

where we denote $N_{C_i^k}$ the number of client in C_i^k.

The following lemma presents an updating law for searching the optimal location and its convergence,

Lemma 1. *Given the initial robot positions $\mathcal{P}^0$ and the clients $\mathcal{D}$, if robot optimal positions are updated according to*

$$\mathcal{P}^k = \bar{\mathcal{P}}^k$$

then the objective function H in (4) is monotonically decreasing, that is

$$H(\mathcal{P}^{k+1}) \leq H(\mathcal{P}^k)$$

Proof. Given the initial robot positions $\mathcal{P}^0$, we can derive the updating chain according to (5)–(6):

$$\mathcal{P}^0 \Rightarrow C^0 \Rightarrow \mathcal{P}^1 \Rightarrow C^1 \Rightarrow \cdots$$

Firstly, we show that the centre $\bar{\boldsymbol{p}}_i^{k+1}$ is the closest point to all clustered clients $\forall \boldsymbol{x}_j \in C_i^k$ once a cluster C_i^k is determined, that is $H(\mathcal{P}^k, C^k) \geq H(\mathcal{P}^{k+1}, C^k)$. Assuming there is an arbitrary position $z \in \boldsymbol{F}$, then the overall distance of the cluster C_i^k corresponding to ith robot has

$$\begin{aligned}
\sum_{j=1}^{N_{C_i^k}} \|\boldsymbol{x}_j - z\|^2 &= \sum_{j=1}^{N_{C_i^k}} \|(\boldsymbol{x}_j - \bar{\boldsymbol{p}}_i^{k+1}) + (\bar{\boldsymbol{p}}_i^{k+1} - z)\|^2 \\
&= \sum_{j=1}^{N_{C_i^k}} (\|\boldsymbol{x}_j - \bar{\boldsymbol{p}}_i^{k+1}\|^2 + \|\bar{\boldsymbol{p}}_i^{k+1} - z\|^2 \\
&\quad + 2(\boldsymbol{x}_j - \bar{\boldsymbol{p}}_i^{k+1})^T (\bar{\boldsymbol{p}}_i^{k+1} - z)) \\
&= \sum_{j=1}^{N_{C_i^k}} \|\boldsymbol{x}_j - \bar{\boldsymbol{p}}_i^{k+1}\|^2 + \sum_{j=1}^{N_{C_i^k}} \|\bar{\boldsymbol{p}}_i^{k+1} - z\|^2 \\
&\quad + \underbrace{2 \sum_{j=1}^{N_{C_i^k}} \left(\boldsymbol{x}_j^T \bar{\boldsymbol{p}}_i^{k+1} - \left(\bar{\boldsymbol{p}}_i^{k+1}\right)^T \bar{\boldsymbol{p}}_i^{k+1} - \boldsymbol{x}_j^T z + \left(\bar{\boldsymbol{p}}_i^{k+1}\right)^T z\right)}_{\text{Term A}}
\end{aligned} \tag{7}$$

Recalling the definition of $\bar{\boldsymbol{p}}_i^k$ in (6), term A can be reformulated as

$$\begin{aligned}
\text{Term A} = 2\Bigg[& N_{C_i^k} \left(\bar{\boldsymbol{p}}_i^{k+1}\right)^T \bar{\boldsymbol{p}}_i^{k+1} - N_{C_i^k} \left(\bar{\boldsymbol{p}}_i^{k+1}\right)^T \bar{\boldsymbol{p}}_i^{k+1} \\
& - N_{C_i^k} \left(\bar{\boldsymbol{p}}_i^{k+1}\right)^T z + N_{C_i^k} \left(\bar{\boldsymbol{p}}_i^{k+1}\right)^T z \Bigg] \\
= 0 &
\end{aligned} \tag{8}$$

Therefore, we can get

$$\sum_{j=1}^{N_{C_i^k}} \|\boldsymbol{x}_j - z\|^2 \geq \sum_{j=1}^{N_{C_i^k}} \|\boldsymbol{x}_j - \bar{\boldsymbol{p}}_i^{k+1}\|^2$$

where equality holds if and only if $z = \bar{\boldsymbol{p}}_i^{k+1}$. Since the robot has a constant traversing velocity λ_i, the objective function $H(\mathcal{P})$ also achieves its minimum at $z = \mathcal{P}^{k+1}$. Therefore we can conclude that

$$H(\mathcal{P}^{k+1}, C^k) \leq H(\mathcal{P}^k, C^k) \tag{9}$$

Next, we consider the case where there exists client $\boldsymbol{x}_i \in \mathcal{D}$ such that different robots between two consecutive iterations cluster it to compare the objectives $H(\mathcal{P}^{k+1}, \mathcal{C}^k)$ and $H(\mathcal{P}^{k+1}, \mathcal{C}^{k+1})$. Let's denote the set of such clients by

$$\hat{\mathcal{D}} = \{\boldsymbol{x}_i \in \mathcal{D} | \boldsymbol{x}_i \in C^k_{i_p}, \boldsymbol{x}_i \in C^{k+1}_{i_n}, i_p \neq i_n\}$$

where i_p and i_n represent the index of clustering robots at k and $k+1$ iteration of the client $\boldsymbol{x}_i$.

Then subtracting two consecutive objective functions yields.

$$\begin{aligned} & H(\mathcal{P}^{k+1}, \mathcal{C}^{k+1}) - H(\mathcal{P}^{k+1}, \mathcal{C}^k) \\ & = \sum_{\boldsymbol{x}_i \in \hat{\mathcal{D}}} \left[\frac{\|\boldsymbol{x}_i - \boldsymbol{p}^{k+1}_{i_n}\|}{\lambda_{i_n}} - \frac{\|\boldsymbol{x}_i - \boldsymbol{p}^{k+1}_{i_p}\|}{\lambda_{i_p}} \right] \end{aligned} \tag{10}$$

According to the definition in (5),

$$\frac{\|\boldsymbol{x}_i - \boldsymbol{p}^{k+1}_{i_n}\|}{\lambda_{i_n}} \leq \frac{\|\boldsymbol{x}_i - \boldsymbol{p}^{k+1}_{i_p}\|}{\lambda_{i_p}}, \forall i \in \hat{\mathcal{D}}$$

Therefore we can derive

$$H(\mathcal{P}^{k+1}, \mathcal{C}^{k+1}) \leq H(\mathcal{P}^{k+1}, \mathcal{C}^k) \tag{11}$$

Combing the results in (9) and (11) proves Lemma 1. □

By comparing with the work in [30], both consider the coverage control for discrete clients and leverage the K-means algorithm to cluster the dominant area for each robot. However, [30] assumes a point-mass model and identical kinematics of robots. The coverage objective can only represent the spatial services that is, only the relative distances between robots and their dominant clients are considered as the objective. This paper, however, studies the coverage control for temporal service, and the non-holonomic kinematic model with heterogeneous but constant-velocity constraints is considered.

3.2 | State regulator using fixed-time control law

This subsection details the design of the state regulator by using a fixed-time control law. Regarding point-to-point control of the robot with a constant velocity, the design of a general control method for the angular velocity of the robot will lead to an insufficient angular velocity, leading the driving distance longer, as depicted in Figure 2. As a result, the general control method will degrade the performance of the time-sensitive coverage problem. What is worse, the robot may keep rotating around the destination if its initial position is too close under the constant-velocity constraint.

In order to make the robot efficiently reach the optimal location computed from (5)–(6), the fixed-time control theory is introduced in the following,

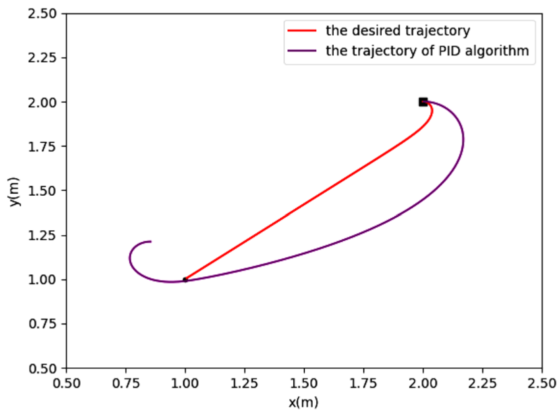


FIGURE 2 The red trajectory is the expected trajectory of the robot from [2,2] to [1,1]. The purple trajectory is the robot driven by the PID controller. The PID controller can not make the robot reach the target point accurately, and the driving trajectory becomes longer, which degrades the performance of the time-sensitive coverage problem.

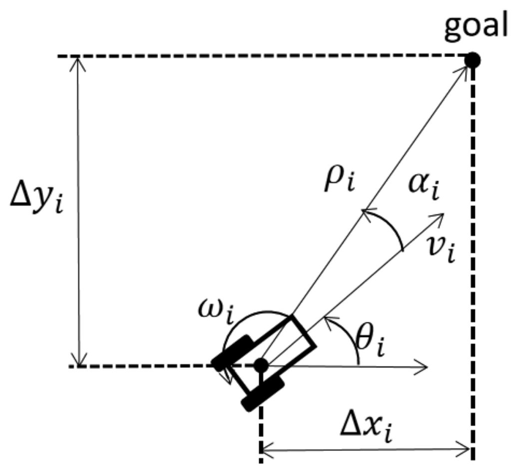


FIGURE 3 Coordinates transformation into polar coordinates.

Lemma 2. *([34]). Suppose there exists a continuously differentiable, positive definite, radially unbounded function V such that*

$$V(x) = 0 \Rightarrow x \in M \tag{12}$$

$$\dot{V}(x(t)) \leq -aV^p(x(t)) - bV^q(x(t)) \tag{13}$$

for all $x \neq 0$, where $a, b > 0$, $0 < p < 1$, $q > 1$ and $M \subset \mathfrak{R}^n$. Then, the system is fixed-time stability, and the time of convergence T is uniformly bounded as $T \leq \frac{1}{a(1-p)} + \frac{1}{b(q-1)}$.

In addition, we reformulate the kinematic (1) under the polar coordinates as shown in Figure 3,

$$\begin{bmatrix} \dot{\rho}_i \\ \dot{\alpha}_i \end{bmatrix} = \begin{bmatrix} -\cos\alpha_i & 0 \\ \frac{\sin\alpha_i}{\rho_i} & -1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \tag{14}$$

where ρ_i and α_i respectively represent the distance from the robot to the target point and the included angle between the velocity direction and the target direction, which can be mathematically denoted by

$$\rho_i = \sqrt{\triangle x_i^2 + \triangle y_i^2}$$
$$\alpha_i = \arctan(\triangle y_i, \triangle x_i) - \theta_i \tag{15}$$

and v_i inherits the constant-velocity constraint in (2). Only the angular velocity ω_i can be controlled.

In order to drive the mobile sensor to $(\rho_i, \alpha_i) = (0, 0)$, we design the state regulator ω_i as follows

$$\omega_i = \phi_i + a\left(\frac{1}{2}\right)^p {\alpha_i}^{2p-1} + b\left(\frac{1}{2}\right)^q {\alpha_i}^{2q-1} + k\alpha_i \tag{16}$$

where

$$\phi_i = \frac{\sin\alpha_i v_i}{\rho_i}$$

Design the Lyapunov function as follows

$$W_i(\alpha_i) = \frac{1}{2}{\alpha_i}^2 \tag{17}$$

The derivative of (17) is as follows

$$\dot{W}_i(\alpha_i) = \dot{\alpha}_i\alpha_i \tag{18}$$

From (14) can get $\dot{\alpha}_i = \frac{\sin\alpha_i v_i}{\rho_i} - \omega_i$

$$\dot{\alpha}_i\alpha_i = \left(\frac{\sin\alpha_i v_i}{\rho_i} - \omega_i\right)\alpha_i \tag{19}$$

Substituting (16) into (19)

$$\begin{aligned}
\dot{\alpha}_i\alpha_i &= \left(-a\left(\frac{1}{2}\right)^p {\alpha_i}^{2p-1} - b\left(\frac{1}{2}\right)^q {\alpha_i}^{2q-1} - k\alpha_i\right)\alpha_i \\
&= -a\left(\frac{1}{2}\right)^p {\alpha_i}^{2p} - b\left(\frac{1}{2}\right)^q {\alpha_i}^{2q} - k{\alpha_i}^2 \\
&= -a\left(\frac{1}{2}{\alpha_i}^2\right)^p - b\left(\frac{1}{2}{\alpha_i}^2\right)^q - k{\alpha_i}^2 \\
&= -aW_i^p(\alpha_i) - bW_i^q(\alpha_i) - k{\alpha_i}^2 \\
&\le -aW_i^p(\alpha_i) - bW_i^q(\alpha_i)
\end{aligned} \tag{20}$$

We can get

$$\dot{W}_i(\alpha_i) \le -aW_i^p(\alpha_i) - bW_i^q(\alpha_i) \tag{21}$$

According to Lemma 2, it is proved that the controller is fixed-time stable.

We set

$$\delta_k = \eta\frac{\rho_i}{\lambda_i} \tag{22}$$

where $0 < \eta < 1$ is a constant coefficient determining the time step between two consecutive waypoints from the K-means algorithm. The principle of choosing parameters a, b, p, q in (16) need to satisfy the following inequalities:

$$\frac{1}{a(1-p)} + \frac{1}{b(q-1)} \le \delta_k \tag{23}$$

Remark that δ_k is the time-varying convergence time of the angular velocity between two consecutive waypoints from the K-means algorithm, which depends on the distance between two consecutive points ρ_i and the constant coefficient η.

Then we can derive our main results as follows:

Theorem 1. *Given the initial robot positions $\mathcal{P}^0$ and the client distribution $\mathcal{D}$, then*

1. *If the numerical optimizer is designed as (5) and (6), the objective function is monotonically decreasing.*
2. *If the state regulator of each robot is designed as (16) with its coefficients satisfying (23), the non-holonomic robots can reach their desired positions even with constant-velocity constraints.*

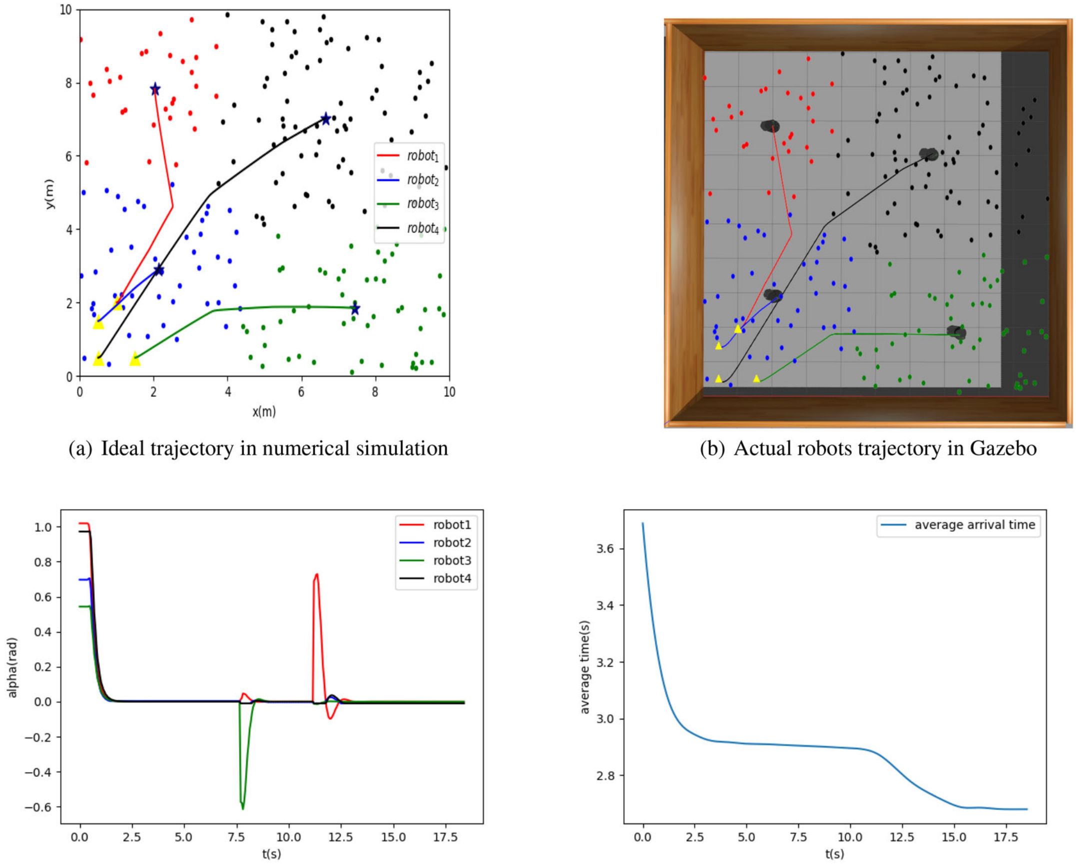
The proof of this theorem is clear according to the Lemma 1 and the analysis above and thus is omitted.

4 | SIMULATIONS

In this part, to prove the effectiveness of our method, we conducted a numerical simulation and Gazebo(physical simulation platform) simulation, respectively. Two hundred clients are randomly generated in a workplace(10 m × 10 m). We set up four mobile robots to cover the clients according to the known position of clients. The initial positions of robots 1 to 4, respectively are $[1, 2], [0.5, 1.5], [1.5, 0.5], [0.5, 0.5]$, as shown in Figure 4(a,b). Their constant velocities, respectively, are [0.5,0.6,0.7,0.8]. Figure 4(a) is the ideal trajectory in numerical simulation, and Figure 4(b) is the actual robot's trajectory in Gazebo.

In Figure 4(a,b), the four yellow triangles represent the robot's initial position, and the red, blue, green, and black lines represent the trajectories of robots 1 to 4. The clients covered by the four robots are respectively red, blue, green, and black points. From Figure 4(a,b), we can see that robot 1, with the smallest velocity of 0.5, covers the fewest clients, while robot 4, with the largest velocity of 0.8, covers the most clients. This indirectly proves the effectiveness of our time-based coverage control method.

Each robot's trajectory passed through three points that got by our numerical optimizer, where robot 1 passes through [1.82,3.33], [2.51,4.58], [2.03,7.82], robot 2 passes through [0.95, 1.88], [1.50,2.40], [2.14,2.91], robot 3 passes through [3.58,1.76], [5.15,1.88], [7.44,1.85], and robot 4 passes through [3.50,4.90], [5.55,6.40], [6.66,7.03]. As can be seen from Figure 4(c), when the robot has a new target point, α of the robot will jump.



(a) Ideal trajectory in numerical simulation

(b) Actual robots trajectory in Gazebo

(c) The changes of α_i for each robots

(d) Average time for robots to reach all clients

FIGURE 4 Virtual robotic simulation results with Gazebo environments.

According to the controller we set for α, α can quickly converge to 0 after each α jump.

In Figure 4(d), the monotonically decreasing curve is the change in the average time for robots to reach all clients covered by themselves as simulation time progresses. According to this curve, we can see that the mobile robots' network coverage performance is gradually improving with the robots moving.

Besides, we also compare our methods with the existing work [28] where a gradient-based controller is proposed for the coverage control of time-sensitive services, and [30] where a planning algorithm based on K-means is proposed for the coverage control of the discrete event. In [28], the clients are assumed to be continuously distributed in the mission space. The comparison is made by evaluating the final objective function of the total minimal arrival time of 50 clients with four robots. Two hundred trials of the previous simulation are done for each method to summarize the statistical characteristics. The comparative results are shown in Figure 5. The blue rectangle in the figure represents the distribution of the results of 200 trials, and the red horizontal line represents the mean value of the results obtained from 200 trials, where our numerical optimizer and state regulator achieve better performance regarding the average arrival time providing by robots' final optimal locations.

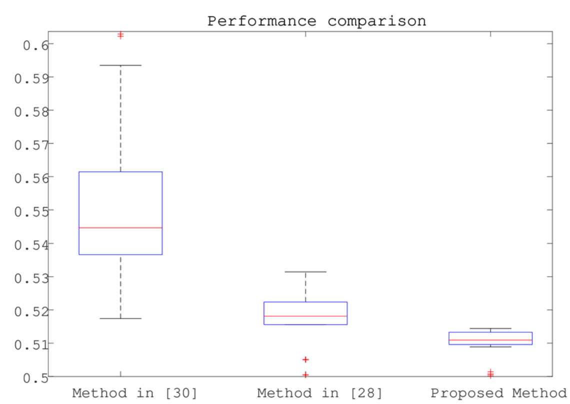
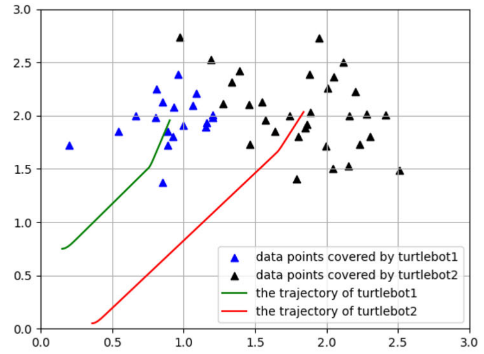


FIGURE 5 The performance comparison among [28, 30] and the proposed methods, where the vertical coordinate is the average time for the robots to reach all clients.

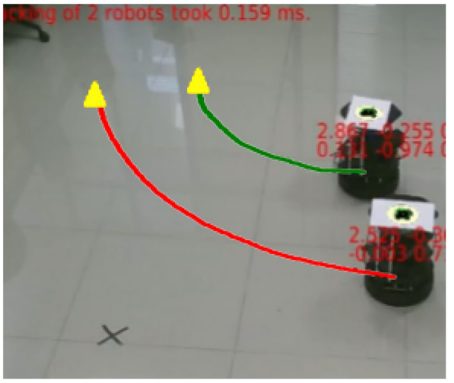
5 | EXPERIMENTS

This experiment develops two real robots covering 50 discrete clients[1]. We set up a camera in our lab and leverage the software WhyCon [35] (WhyCon is a vision-based localization system

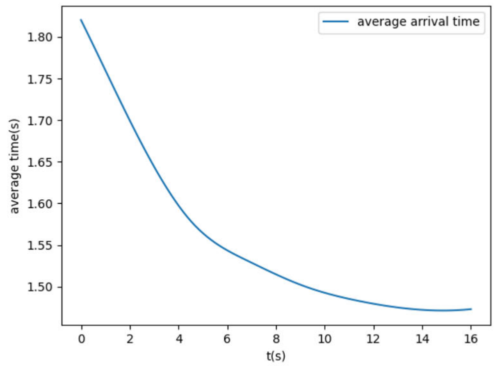
[1] More details can be found from our recorded experimental video from https://youtube/IqwV1cr2sAA



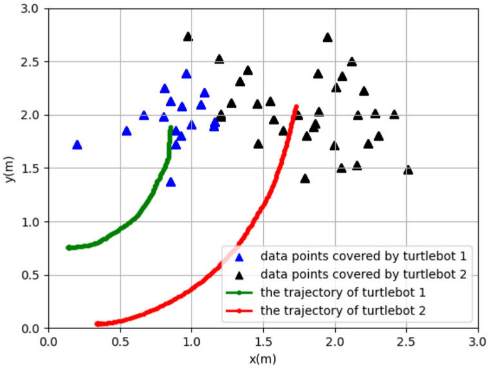
(a) Ideal trajectory in numerical simulation



(b) Actual robots trajectory in experiment



(c) Average time for robots to reach all clients



(d) The real trajectory of the turtlebots

FIGURE 6 Experimental results using two moving turtlebots serving 50 virtual clients.

that can be used with low-cost web cameras and achieves millimetre precision with very high performance) to provide a precise location of robot and create a two-dimensional coordinate system in the field of view of the camera. The robots in Figure 6(b) are turtlebots (turtlebot is a low-cost personal robot kit with open-source software, and its driver is a Kobuki chassis). Each turtlebot has a marker, which the camera can recognize to obtain the real-time position of the turtlebot in the two-dimensional coordinate system.

First, we randomly distribute 50 clients in the 3 m × 3 m and publish the position of each data point to the turtlebots. Then, the initial positions of turtlebots 1 and 2, respectively are [0.15,0.75],[0.36,0.05], as shown in Figure 6(a), and their constant velocities respectively are [0.15,0.2]. Finally, after utilizing the coverage control method we designed, the turtlebots 1 and 2, respectively drove to [0.86,1.91],[1.76,2.08], as shown in Figure 6(b).

Figure 6(c) shows that with the time of the experiment, the average time for turtlebots to reach all clients is covered by themselves. A monotonically decreasing tendency supports our main results in the Theorem 1.

Figure 6(d) shows the real trajectory of the turtlebots, where the green trajectory is turtlebot 1, and the red trajectory is turtlebot 2. The blue data point is covered by turtlebot 1 with a constant velocity of 0.15, and the black data point is covered by turtlebot 2 with a constant velocity of 0.2.

By comparing Figure 6(a,d), we can conclude that in the real environment, there are disturbances and position errors in the driving process of turtlebot, which lead to the difference between the robot's driving route and the ideal driving route, but the final coverage performance is the same, which indicates that our method is also effective in the real environment.

6 | CONCLUSION

This paper presents a time-based coverage control method for non-holonomic mobile robots with heterogeneous and constant velocities. Given that each robot has a fixed constant velocity, the coverage ability of each robot is closely related to its fixed constant velocity. Therefore, each client's service quality is mainly evaluated by the minimal arrival time of robots. Finally, the time-sensitive coverage control problem can be formulated as a two-step process. First, the local optimal coverage control positions were obtained using the time-sensitive K-means algorithm. Second, the angular velocity state regulator we designed controls the robots to drive to the local optimal coverage control position. The simulation and experimental results show that the proposed method is effective for non-holonomic mobile robots with heterogeneous and constant velocities to serve discrete time-sensitive clients.

AUTHOR CONTRIBUTIONS

Liang Zhang: Conceptualization, investigation, methodology, software, supervision, writing - original draft, writing - review and editing. Jinghui Deng: Conceptualization, data curation, investigation, methodology, resources, software, validation, writing - original draft, writing - review and editing. Kun Zhou: Investigation, methodology, software. Tao Yu: Conceptualization, investigation, methodology, supervision. Jun Song: Conceptualization, investigation, methodology, supervision. Shuping He: Conceptualization, formal analysis, funding acquisition, methodology, project administration, supervision, writing - review and editing.

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CONFLICT OF INTEREST STATEMENT

The authors have declared no conflict of interest.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

ORCID

Liang Zhang https://orcid.org/0000-0001-5690-6563
Jinghui Deng https://orcid.org/0000-0002-8858-691X
Shuping He https://orcid.org/0000-0003-1869-2116

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SUPPORTING INFORMATION

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